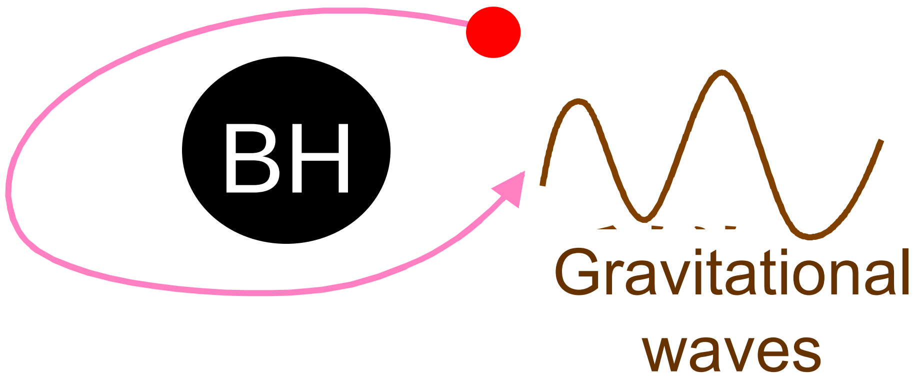
Gravitational waves from Extreme mass ratio inspirals

Gravitational Radiation Reaction Problem



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Various sources of gravitational waves

- Inspiraling binaries
- (Semi-) periodic sources
  - Binaries with large separation (long before coalescence)
    - a large catalogue for binaries with various mass parameters with distance information
  - Pulsars
- Sources correlated with optical counter part
  - supernovae
  - γ - ray burst
- Stochastic background
  - GWs from the early universe
  - Unresolved foreground

Inspiraling binaries

In general, binary inspirals bring information about

- Event rate
- Binary parameters
- Test of GR

- Stellar mass BH/NS
  - Target of ground based detectors
  - NS equation of state
  - Possible correlation with short γ-ray burst
  - primordial BH binaries (BHMACHO)
- Massive/intermediate mass BH binaries
  - Formation history of central super massive BH
- Extreme (intermidiate) mass-ratio inspirals (EMRI)
  - Probe of BH geometry

- Inspiral phase (large separation)

  Clean system (Cutler et al, PRL **70** 2984(1993))

  Negligible effect of internal structure

Accurate prediction of the wave form is requested

- for detection
- for parameter extraction
- for precision test of general relativity

(Berti et al, PRD **71**:084025,2005)

- Merging phase - numerical relativity

  recent progress in handling BHs

- Ringing tail - quasi-normal oscillation of BH

Extreme mass ratio inspirals (EMRI)

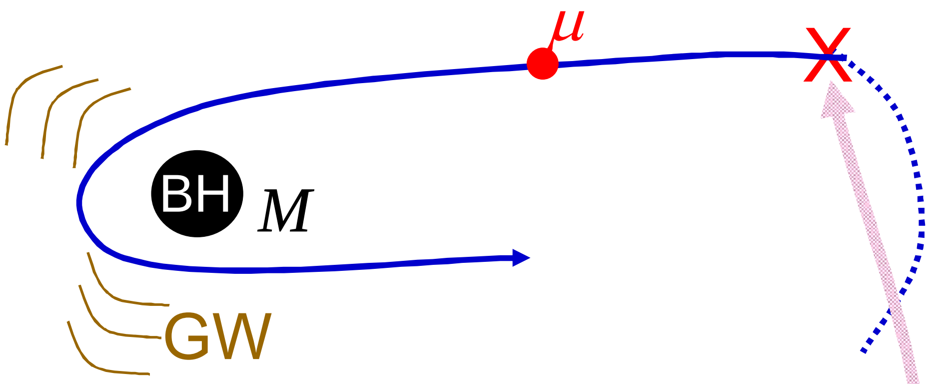
- LISA sources 0.003-0.03Hz

  → merger to $M \sim 10^5 M_{\odot} - 5\times 10^6 M_{\odot}$

  white dwarfs (μ=0.6$M_{\odot}$),

  neutron stars (μ=1.4$M_{\odot}$)

  BHs (μ=10$M_{\odot}$,~100$M_{\odot}$)



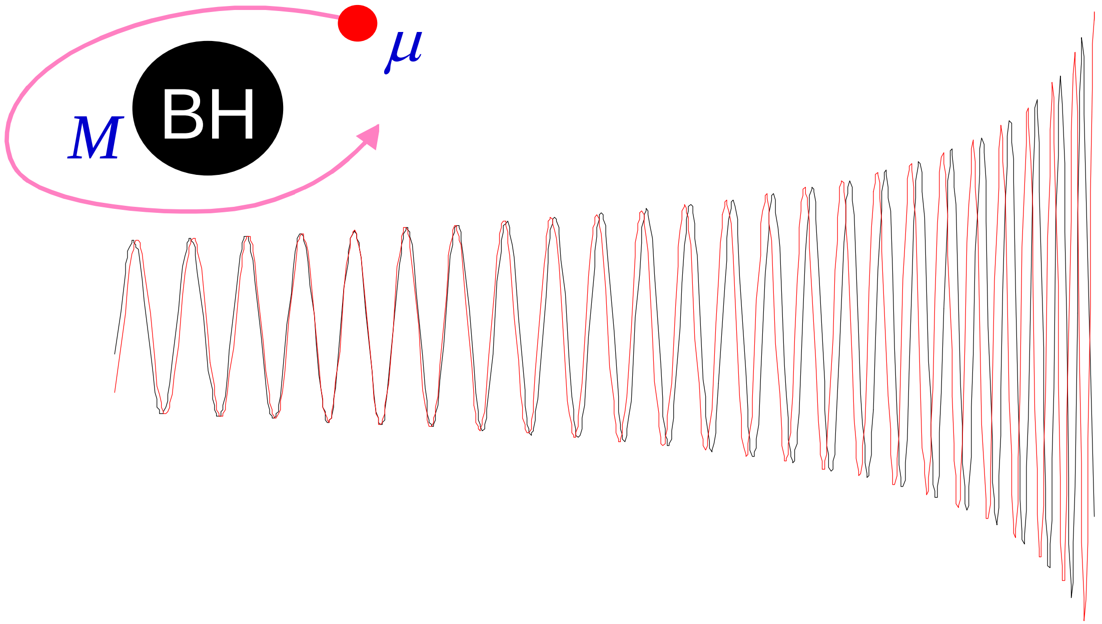
- Formation scenario
  - star cluster is formed
  - large angle scattering encounter put the body into a highly eccentric orbit
  - Capture and circularization due to gravitational radiation reaction *~last three years: eccentricity reduces* $1\text{-}e \rightarrow O(1)$
- Event rate:

  a few ×10^2 events for 3 year observation by LISA (Gair et al, CGQ **21** S1595 (2004))

  although still very uncertain.(Amaro-Seoane et al, astro-ph/0703495)

- $\mu \ll M$ Radiation reaction is weak

  Large number of cycles N before plunge in the strong field region



Roughly speaking, difference in the number of cycle $\Delta N > 1$ is detectable.

- High-precision determination of orbital parameters
- maps of strong field region of spacetime
  - Central BH will be rotating: $a \sim 0.9M$

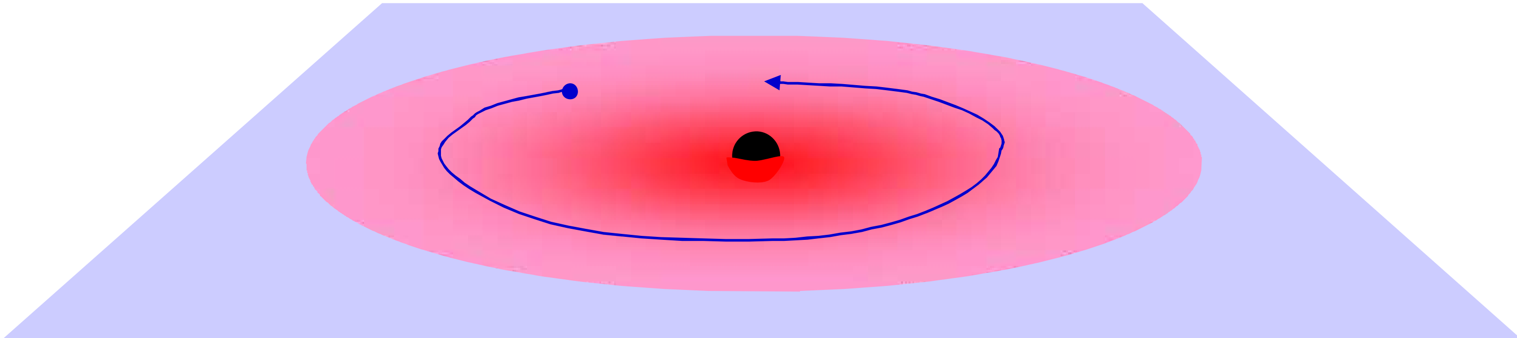
Probably clean system

- Interaction with accretion disk (Narayan, ApJ, **536**, 663 (2000))

,assuming almost spherical accretion (ADAF)

$$t_{df} = \frac{v_{rel}^3}{4\pi \log \Lambda G^2 m_{satellite} \rho}$$

$$\approx 4.5\times 10^{12} \frac{M}{10^6 M_\odot} \left(\frac{\mu}{10 M_\odot}\right)^{-1} \left(\frac{\dot{m}}{10^{-2} \dot{M}_{Edd}}\right)^{-1} \mathrm{yr}$$



Frequency shift due to interaction

$$\frac{\Delta f}{f} \approx \frac{T_{obs}}{t_{df}}$$

obs. period $\sim 1\mathrm{yr}$

Change in number of cycles

$$\Delta N \approx \Delta f\, T_{obs} \approx f T_{obs} \frac{T_{obs}}{t_{df}}$$

Theoretical prediction of Wave form

Template in Fourier space

$$\boxed{h(f) \approx A f^{-7/6} e^{i\Psi(f)}} \qquad \boxed{A = \frac{1}{\sqrt{20\pi^3}} \frac{\mathcal{M}^{5/6}}{D_L}, \quad \mathcal{M} = \mu^{3/5} M^{2/5}, \quad \eta = \frac{\mu}{M}}$$

$$\Psi = 2\pi f t_c - \phi_c + \frac{3}{128} (\pi \mathcal{M} f)^{-5/3} \left[1 + \underbrace{\frac{20}{9} \left(\frac{743}{331} + \frac{11}{4}\eta \right) u^{2/3}}_{\text{1PN}} - \underbrace{(16\pi - \beta) u}_{\text{1.5PN}} + \cdots \right]$$

$$u \equiv \pi M f = O(v^3)$$

for quasi-circular orbit

- We know how higher expansion proceeds.

  $\Rightarrow$Only for detection,
  higher order template may not be necessary?

- We need higher order accurate template
  for precise measurement of parameters (or test of GR).

c.f. observational error in parameter estimate $\propto$ signal to noise ratio

Test of GR

Effect of modified gravity theory

Scalar-tensor type

Mass of graviton

$$\Psi = \cdots + \frac{3}{128}(\pi \mathcal{M} f)^{-5/3}\left[\alpha u^{-2/3} + 1 + \left(\frac{3715}{756} + \frac{55}{9}\eta + \frac{128}{3}\eta\beta_g\right)u^{2/3} - (16\pi - \beta)u + \cdots\right]$$

$$\alpha \propto \frac{1}{\omega_{BD}}$$

Dipole radiation $= -1$ PN

$$\beta_g = \frac{\pi^2 M}{\lambda_g^2}\int a^2 d\eta$$

$$u = \pi M f = O(v^3)$$

Current constraint on dipole radiation:

$\omega_{BD} > 140$, (600)

4U 1820-30 (NS-WD in NGC6624)

(Will & Zaglauer, ApJ **346** 366 (1989))

Constraint from future observation:

LISA– $1.4M_{\odot}$NS+$400M_{\odot}$BH: $\omega_{BD} > 2\times10^4$

(Berti & Will, PRD**71** 084025(2005))

Decigo–$1.4M_{\odot}$NS+$10M_{\odot}$BH: $\omega_{BD} > 5\times10^9$?

Constraint from future observation:

LISA– $10^7M_{\odot}$BH+$10^7M_{\odot}$BH:

graviton compton wavelength

$\lambda_g >$ 1kpc

(Berti & Will, PRD**71** 084025(2005))

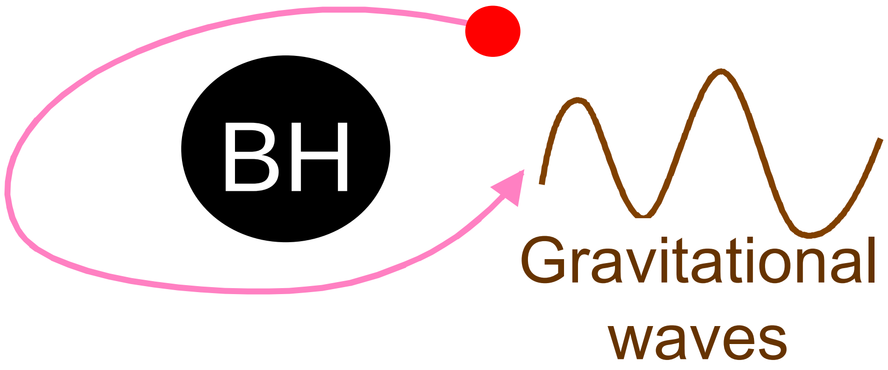
Black hole perturbation

- $G^{\mu\nu}[\mathbf{g}] = 8\pi G T^{\mu\nu}$

$$g_{\mu\nu} = g^{BH}_{\mu\nu} + h^{(1)}_{\mu\nu} + h^{(2)}_{\mu\nu} + \cdots$$

✧ $M \gg \mu$

✧ v/c can be $O(1)$



Linear perturbation

$$\delta G^{\mu\nu}[\mathbf{h}^{(1)}] = 8\pi G T^{(1)\mu\nu}$$

$\Rightarrow L\psi^{(1)} = 4\pi\sqrt{-g}\,T^{(1)}$:master equation

Regge-Wheeler formalism (Schwarzschild)
Teukolsky formalism (Kerr)

Mano-Takasugi-Suzuki's method (systematic PN expansion)

Teukolsky formalism

Teukolsky equation

$$\boxed{L\,\psi = 4\pi\sqrt{-g}\,T}$$

$T = \tau_{\mu\nu} T^{\mu\nu}$

$\underline{\tau_{\mu\nu}}$: 2nd order differential operator

ψ: projected Weyl curvature

First we solve homogeneous equation

$\Lambda \equiv \ell, m, \omega$

$$\boxed{L\Omega = 0} \qquad \Omega = \sum_{\Lambda} R_{\Lambda}(r) \underline{Y_{\Lambda}(\theta,\varphi)} e^{-i\omega t}$$

$$\left[\partial_r^2 + \cdots\right] R(r) = 0$$

$Y_{\Lambda}(\theta,\varphi)$: Angular harmonic function

Construct solution using Green fn. method.

$$\psi = \sum_{\Lambda} \Omega_{\Lambda}^{up}(x) Z_{\Lambda} \approx \sum_{\Lambda} \Omega_{\Lambda}^{up}(x) \frac{1}{{}_s W_{\Lambda}} \int \sqrt{-g}\, d^4x'\, R_{\Lambda}^{in}(r') \overline{Y}_{\Lambda}(\theta',\varphi') e^{i\omega t} T(x') + \cdots$$

Wronskian ${}_s W_{\Lambda} \approx {}_s R_{\Lambda}^{up} \overleftrightarrow{\partial_r}\, {}_s R_{\Lambda}^{in}$

at $r \to \infty$

$$\boxed{\psi \sim \frac{1}{2}\left(\ddot{h}_+ - i\,\ddot{h}_\times\right)}$$

$$\frac{dE}{dt} = -\sum_{\Lambda} \left|Z_{\Lambda}\right|^2$$: energy loss rate

$$\frac{dL_z}{dt} = -\sum_{\Lambda} \frac{m}{\omega} \left|Z_{\Lambda}\right|^2$$: angular momentum loss rate

Leading order wave form

Energy balance argument is sufficient.

$$\boxed{\frac{dE_{GW}}{dt} = -\frac{dE_{orbit}}{dt}}$$

Wave form $\equiv \dfrac{df}{dt}$ for quasi-circular orbits, for example.

$$\frac{df}{dt} = \frac{dE_{orbit}}{dt} \Big/ \frac{dE_{orbit}}{df}$$

leading order

$$\frac{dE_{orbit}}{dt} = 0 + O(\mu) + O(\mu^2)$$

$$\frac{dE_{orbit}}{df} = (\text{geodesic}) + O(\mu) + O(\mu^2)$$

self-force effect

Radiation reaction for General orbits in Kerr black hole background

- Radiation reaction to the Carter constant

Schwarzschild "constants of motion" $E, L_i \Leftrightarrow$ Killing vector

Conserved current for GW corresponding to Killing vector exists.

$$E_{GW} = \int d\Sigma^{\mu} t_{\mu\nu}^{(GW)} \xi^{\nu}$$

$$\dot{E}_{orbit} = -\dot{E}_{gw}$$ In total, conservation law holds.

Kerr conserved quantities $E, L_z \Leftrightarrow$ Killing vector

$Q \not\Leftrightarrow$ Killing vector

- We need to directly evaluate the self-force acting on the particle, but it is divergent in a naïve sense.

Adiabatic approximation for Q,

which differs from energy balance argument.

- orbital period << timescale of radiation reaction
- It was proven that we can compute the self-force using the radiative field, instead of the retarded field, to calculated the long time average of $\dot{E}, \dot{L}_z, \dot{Q}$.

(Mino Phys. Rev. **D67** 084027 ('03))

$$h_{\mu\nu}^{(rad)} = \left[h_{\mu\nu}^{(ret)} - h_{\mu\nu}^{(adv)} \right] / 2$$:radiative field

At the lowest order, we assume that the trajectory of a particle is given by a geodesic specified by E, L_z, Q.

$$\boxed{\left\langle \dot{Q} \right\rangle = \frac{1}{\mu} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} d\tau \, \frac{\partial Q}{\partial u^{\alpha}} F^{\alpha}\left[h_{\mu\nu}^{(rad)} \right]}$$

Radiative field is not divergent at the location of the particle. $\Rightarrow$ Regularization of the self-force is unnecessary!

Simplified *dQ/dt* formula

(Sago, Tanaka, Hikida, Nakano, Prog. Theor. Phys. **114** 509('05))

- Self-force f^{α} is explicitly expressed in terms of $h_{\mu\nu}$ as

$$f^{\alpha} = -\frac{1}{2}\left(g^{\alpha\beta} + u^{\alpha}u^{\beta}\right)\underline{\left(h_{\beta\gamma;\delta} + h_{\beta\delta;\gamma} - h_{\gamma\delta;\beta}\right)}u^{\gamma}u^{\delta}$$

$$\frac{dQ}{d\tau} = 2K^{\nu}_{\mu}u^{\mu}f_{\nu}$$

Killing tensor associated with Q

$$Q \equiv K^{\nu}_{\mu}u^{\mu}u_{\nu}$$

$$\boxed{h_{\mu\nu} = \tau^{*}_{\mu\nu}\ {}_{s}\Psi}$$

Complicated operation is necessary for metric reconstruction from the master variable.

after several non-trivial manipulations

- We arrived at an extremely simple formula:

$$\left\langle \frac{dQ}{dt} \right\rangle = 2\left\langle \frac{\left(r^2+a^2\right)P(r)}{\Delta} \right\rangle \left\langle \frac{dE}{dt} \right\rangle - 2\left\langle \frac{aP(r)}{\Delta} \right\rangle \left\langle \frac{dL}{dt} \right\rangle + 2 \sum_{l,m,\omega=\omega_{l,m}^{n_r,n_\theta}} \frac{n_r\Omega_r}{\omega}\left|Z_{l,m,\omega}\right|^2$$

Only discrete Fourier components exist

$$\omega = \omega_m^{n_r,n_\theta} \approx \left(m\Omega_\varphi + n_r\Omega_r + n_\theta\Omega_\theta\right)$$

$$\boxed{P(r) = E\left(r^2+a^2\right) - aL}$$

$$\boxed{\Delta = r^2 - 2Mr + a^2}$$

■Use of systematic PN expansion of BH perturbation.
■Small eccentricity expansion
■General inclination

$$\left\langle \frac{d\mathcal{C}}{dt} \right\rangle = -\frac{64}{5}\left(\frac{\mu}{M^2}\right) M^2 v^6 (1-e^2)^{3/2} \left(1-Y^2\right) \left[\left(1+\frac{7}{8}e^2\right) - \left(\frac{743}{336} - \frac{23}{42}e^2\right) v^2 - \left(\frac{85Y}{8} + \frac{211Y}{8}e^2\right) qv^3 + \left(4 + \frac{97}{8}e^2\right) \pi v^3 - \left(\frac{129193}{18144} + \frac{84035}{1728}e^2\right) v^4 - \left(\frac{329}{96} - \frac{53Y^2}{8} + \left\{\frac{929}{96} - \frac{163Y^2}{8}\right\} e^2\right) q^2 v^4 + \left(\frac{2553Y}{224} - \frac{553Y}{192}e^2\right) qv^5 - \left(\frac{4159}{672} + \frac{21229}{1344}e^2\right) \pi v^5\right]$$

(Ganz, Hikida, Nakano, Sago, Tanaka, Prog. Theor. Phys. ('07))

Summary

Among various sources of GWs, E(I)MRI is the best for the test of GR.

For high-precision test of GR, we need accurate theoretical prediction of the wave form.

Adiabatic radiation reaction for the Carter constant has been computed.

leading order

second order

$$\frac{dE_{orbit}}{dt} = 0 + \boxed{O(\mu)} + O(\mu^2)$$

$$\frac{dE_{orbit}}{df} = (\text{geodesic}) + \boxed{O(\mu)} + O(\mu^2)$$

Direct computation of the self-force at O(μ) is also almost ready in principle.

However, to go to the second order, we also need to evaluate the second order self-force.