New Calculation of Cosmic-Ray Antiproton and Positron Flux

By
Tadao Mitsui

Department of Physics, School of Science,
University of Tokyo

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Abstract

Cosmic-ray antiproton ($\bar{p}$) and positron ($e^+$) fluxes were calculated using the Standard Leaky Box (SLB) and Diffusion and Diffusive Reacceleration (DR) models with new interaction cross sections and a new path-length in the Galaxy. The results of the SLB and Diffusion models agree with the recent observations of $\bar{p}$ and $e^+$ by BESS and HEAT experiments, respectively. The results of the DR model did not agree with the data, although they were consistent with the data within errors. Low-energy $\bar{p}$ flux arising from evaporating primordial black holes (PBHs) was also investigated using a 3-D Monte Carlo simulation based on the Diffusion model. This flux was used with recent observations to derive new upper limits on (i) the local PBH explosion rate $\mathcal{R} < 1.3 \times 10^{-2} \text{pc}^{-3}\text{yr}^{-1}$, (ii) the fraction of the Universe’s mass going into PBHs with particular mass, and (iii) the average density of PBHs in the Universe.
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Chapter 1

Introduction

The origins of cosmic-ray antiprotons (\(\bar{p}\)'s) and positrons (\(e^+\)) are of great physical interest. Possible sources of these antiparticles are (i) high energy collisions of cosmic-ray protons (and nuclei) with interstellar medium (ISM) [1, 2, 3, 4, 5] ("secondary" \(\bar{p}\)'s/positrons), and (ii) unknown physical processes emitting particles and antiparticles in an equal amount, such as the evaporation of primordial black holes (PBHs) [6, 7] and annihilation of supersymmetric dark matter particles [8].

The expected flux of the secondary \(\bar{p}\)'s and positrons is, in principle, calculable because (i) the production of secondary species in our Galaxy has been confirmed by the observation of light nuclei in cosmic rays such as boron and beryllium that are rarely produced by nucleosynthesis in stars, and (ii) the production cross sections of \(\bar{p}\)'s and positrons by \(pp\) (nuclei) collisions are also known. By comparing the calculated and observed fluxes of \(\bar{p}\) and \(e^+\), cosmic-ray propagation models can be tested. Because \(\bar{p}\) and \(e^+\) are mainly proton secondaries, we can trace the propagation history of the dominant proton component, although secondary nuclei, such as boron and beryllium, provide information of heavier nuclei only.

Low-energy antiproton flux is also sensitive to novel primary sources such as evaporating PBHs because, although the kinematics of secondary \(\bar{p}\) production by \(pp\) collisions should lead to a steep drop in the resultant \(\bar{p}\) flux at kinetic energies less than 2 GeV, the expected flux of \(\bar{p}\)'s from PBHs (PBH-\(\bar{p}\)'s) has contrastingly been shown to increase with decreasing kinetic energy down to \(\sim 0.2\) GeV [6], thus providing a distinct signature below 1 GeV. In addition, PBH-\(\bar{p}\)'s are stored in the Galaxy for \(\sim 10^8\) yr [10] because of the turbulent magnetic field in the Galaxy. This leads to an enhancement factor of \(\sim 10^3\) compared with \(\gamma\)-rays. Such enhancement, however, is dependent on cosmic-ray propagation models, i.e., the storage time in the Galaxy and local flux in the vicinity of the Solar system. Therefore, the study of propagation models using secondary \(\bar{p}\)'s and positrons is also important in the search for evaporating PBHs using the \(\bar{p}\) channel. Such analyses are possible because the fluxes of PBH- and secondary \(\bar{p}\)'s have different spectral shapes as describe above.
Figure 1.1: Observations of cosmic-ray \( \bar{p}/p \) ratios and predictions under the SLB model. Data points are from the BESS experiment [13] (filled squares), IMAX experiment [14] (filled circles), PBAR experiment [16] (open diamonds), LEAP experiment [17, 18] (open circles), Buffington et al. [19] (open square), Bogolomov et al. [20, 21] (open triangles), and Golden et al. [22] (filled triangles). SLB predictions are from Gaisser et al. [1] (solid line) and Webber et al. [2] (dashed line). For solar modulation, see Appendix B.
Figure 1.2: Observations of cosmic-ray $e^+/ (e^- + e^+)$ flux ratio and prediction under the Diffusion model. Data points are from Fanselow et al. [25], Daugherty et al. [26], Buffington et al. [27], Muller et al. [28], Golden et al. [29], HEAT experiment [30], and AESOP experiment [31]. The prediction of the Diffusion model is from Protheroe [5].
Recently, there has been great progress in measuring cosmic-ray antiprotons at low energies. Antiprotons having energies from 175 to 500 MeV were first observed by the Balloon-borne Experiment with a Superconducting magnet rigidity Spectrometer (BESS) [11, 12, 13] (filled square, Fig. 1.1 shown in \( \bar{p}/p \) flux ratio). Although a number of authors have calculated the expected secondary \( \bar{p} \) flux [1, 2, 3, 4] based on the standard cosmic-ray propagation theory, i.e., the Standard Leaky Box (SLB) model (see Chapter 2), observational sensitivities have not reached the predicted level at energy ranges below 500 MeV until recently. Actually the upper limits of \( \bar{p}/p \) ratio obtained by LEAP [17, 18] and PBAR [16] experiments (open circles and open diamonds respectively in Fig. 1.1) are consistent with the predictions by Gaisser et al. [1] or Webber et al. [2] (solid and dashed lines, Fig. 1.1).

Low-energy \( \bar{p} \)'s observed by the BESS experiment were at a level slightly below the upper limit of LEAP experiment. It is of great interest whether or not these \( \bar{p} \)'s can be explained by the SLB model. Since this model was established to explain the data of cosmic-ray nuclei, it is not observationally known whether or not the propagation of protons can be explained with this model unless it is tested by the proton secondary component, i.e., \( \bar{p} \) and \( e^+ \). If the observed \( \bar{p} \) and \( e^+ \) fluxes agree with the predictions of the SLB model, it means that the parent protons traverse an equal amount of ISM as is traversed by nuclei. This further indicates that the sources (acceleration regions) of protons are the same as those of nuclei. This is very important information because, although the supernova remnant (SNR) is considered to be a plausible candidate of cosmic-ray source by the analysis of the source abundance ratio of nuclei, the source of dominant protons can be, in principle, different from the nuclei source.

However, there exist several conflicting results in \( \bar{p} \) flux prediction as shown in Fig. 1.1 (details are described in Chapter 3). The \( \bar{p} \) flux calculated by Webber et al. [2] is larger than the upper limit of Gaisser et al. [1]. It is necessary to perform new calculations using the latest input data, such as interaction cross sections and escape length, and the causes of the discrepancy should be explained in order to interpret the recent observations by the BESS experiment.

Other propagation models can be tested by using \( \bar{p} \) flux. The energy spectrum of the secondary \( \bar{p} \)'s is expected to drop below the kinetic energy of 2 GeV, in contrast with the other cosmic-ray components whose energy spectra steeply drop off with increasing energy. Because of this unique spectrum, \( \bar{p} \)'s are sensitive to the propagation processes in the Galaxy, especially reacceleration, a process not considered under the SLB model. In a recent paper [23, 24], Heinbach and Simon proposed the “Diffusive Reacceleration model” (DR model), where they showed that a second-order Fermi type of acceleration can naturally occur during the diffusive propagation in the Galaxy. Under these conditions, the escape length \( \lambda_{esc}(E) \), which defines particle paths in the Galaxy, becomes a power-law of rigidity with an exponent of \(-1/3\), which would indicate a Kolmogorov-type spectrum of magnetic turbulence. Since both DR and SLB models fit the observational data of cosmic-ray nuclei [23, 24], these models should be further tested by other components. In Chapter 4, we compare the \( \bar{p} \) spectrum calculated using SLB and DR models with the recent observations.
In Chapter 6, we examine whether the signature of evaporating PBHs exists in low-energy antiproton data. Even negative results could lead to a novel density constraint on PBHs in the vicinity of the Solar system.

Positrons (e+) are also mainly proton secondaries, but more abundant than antiprotons because of the low threshold of e+ production in pp collisions in interstellar space. As a result, a number of observations have succeeded in detecting cosmic-ray e+ [25, 26, 27, 28, 29, 30, 31] as shown in Fig. 1.2 (shown in “e+ fraction,” i.e., the flux ratio e+/(e− + e+)).

Those data between ∼1 to several GeV are in agreement with the calculations of Protheroe [5] (solid line, Fig. 1.2) using the Diffusion model, which is considered to be a standard propagation model for electrons and positrons (see Chapter 2). Below 1 GeV the observed e+ fraction is higher than the predicted one, but no conclusion can be drawn because the prediction [5] does not include the solar modulation effect. On the other hand, the excess of e+ fraction above 10 GeV reported by several experiments [27, 28, 29] seems to be significant since solar modulation does not affect flux at this energy region (Appendix B). These results have inspired a variety of interpretations involving either a depletion of the primary electron source at high energy or new sources of e± pairs such as the annihilation of hypothetical dark matter particles [8] or pair creation in the strong magnetic field at pulsars [9], although they offer no definitive conclusions.

In a recent observation by the HEAT experiment [30], however, no evidence of this rise in e+ fraction was seen at energies above 10 GeV (filled square, Fig. 1.2). In the HEAT experiment, a high degree of hadron rejection was achieved by using a magnet spectrometer combined with a transition radiation detector, an electromagnetic calorimeter, and time-of-flight counters. None of the other experiments employed such techniques. Here we confine ourselves to interpretations of this new measurement.

Although the rise was not seen, the data of HEAT experiment are still higher than the predicted curve by Protheroe [5]. However, after the work of Protheroe, it was found by the HEAO-3 experiment [32] that the path-length of cosmic rays in the Galaxy is longer than previously considered. The e+ fraction is consequently expected to be larger than that shown in the Fig. 1.2 because e+ are secondaries produced in traversing ISM while e− are mainly primaries. We perform new calculations of the e+ fraction using the latest path-length data and based on the Diffusion model (Chapter 5). The e+ fraction is also calculated was the DR model.
Chapter 2
Cosmic-Ray Propagation Models

In this chapter, cosmic-ray propagation models are reviewed and the data of nuclei fluxes are analyzed in order to choose appropriate propagation models and to tune parameters of the models, such as escape length.

2.1 Leaky Box and Diffusion Models

Most of cosmic-ray nuclei data are explained by the Leaky Box (LB) propagation model [32, 33, 34, 35, 36, 37, 38, 39]. In the LB model, one assumes that cosmic-ray particles are confined in the Galaxy for a certain period and then slowly escape to intergalactic space. The spatial variations of the densities of cosmic rays, their sources, and ISM are neglected. The escape process of cosmic rays is assumed to be a Poisson one, so that the age distribution of cosmic rays is exponential. The path-length distribution (PLD) of cosmic rays also becomes exponential, because the density of ISM is considered to be homogenized in the confinement volume. Such exponential PLD fits the relative abundances of elements [32, 33, 34, 35, 36] and isotopes [37, 38, 39].

Generally, cosmic rays are assumed under the LB model to be “preaccelerated” at the source regions and no “reacceleration” is considered to occur during propagation in the Galaxy. Under such conditions, the relative abundances of cosmic-ray nuclei at a given energy per nucleon are dependent only on the relative source abundances and escape length $\lambda_{esc}$ at that energy per nucleon, where $\lambda_{esc} = \lambda_{esc}(E)$ is the mean path-length that cosmic rays traverse until they escape from the Galaxy (i.e., the mean path-length averaged over PLD). The escape length is then determined by the relative abundances of cosmic-ray nuclei for each energy per nucleon (or rigidity). This energy-dependent escape length fits the power-law of rigidity with an exponent of $\sim 0.6$ above $\sim 5$ GV [32, 34, 36]. This LB model without reacceleration and other additional processes is generally referred to as the “Standard Leaky Box (SLB) model.”

Although the LB model is a useful approximation, the actual propagation process is understood as a diffusive process where the magnetic field irregularities serve as a scattering center (Diffusion model). In the Diffusion model, cosmic-ray sources are located in a thin galactic disk of thickness $2h_g \simeq 200$ pc. Cosmic rays released from
sources experience the diffusive propagation process in the turbulent magnetic field of the Galaxy with the diffusion coefficient $D \sim 10^{28} \text{ cm}^2 \text{s}^{-1}$ [40], which corresponds to a mean free path $\sim \text{pc}$. Then they diffuse out of the disk into the larger galactic halo of thickness $h_h \sim \text{a few kpc}$, where the diffusion coefficient $D$ is assumed to be equal to that in the disk. When cosmic rays reach the boundary of the halo, they escape out of the Galaxy.

The relation of the LB and Diffusion models were discussed by Ptuskin et al. [10, 41, 42], and Nishimura et al. [43], where the LB model was considered to be a first-order approximation of the Diffusion model as shown in Appendix C. This approximation is valid for particles whose interactions (including the energy changing process) are slow compared with diffusion process, e.g., stable nuclei, protons, and antiprotons. Since these particles traverse the galactic plane, where the Solar system is located, many times before escape or interactions, flux near the Solar system is not affected by the spatial variations of densities of cosmic rays, their sources, and ISM. On the other hand, one should use the Diffusion model for particles whose interactions are fast and occur in the halo as well as in the disk, e.g., radioactive beryllium nuclei $^{10}\text{Be}$ and electrons/positrons. Since the half life of $^{10}\text{Be}$ is $1.5 \times 10^6 \text{ yr}$ while the confinement times of cosmic rays in the disk and halo are $\sim 10^7$ and $\sim 10^8 \text{ yr}$ respectively [10, 41], $^{10}\text{Be}$ nuclei will decay before they diffuse to the boundary of halo or even before they diffuse out of the disk, so that the spatial variations of cosmic-ray densities is important for the calculations of flux near the Solar system [41].

2.2 Other Models

In the Dynamical Halo model [44, 40], the convection of cosmic rays due to the galactic wind is considered. In this model the flow of cosmic rays in the halo is faster than that in the simple Diffusion model due to this convection effect. Consequently, a larger halo is required to confine cosmic rays for the same period as that in the Diffusion model. In a recent paper, however, Webber et al. [40] set upper limits on both the halo size and convection velocity to be less than 4 kpc and 20 km/s respectively. They also showed that the main process of carrying cosmic rays away from the galactic plane is diffusion and that convection does not play an important role. Therefore we will not consider the Dynamical Halo model here.

The Nested Leaky Box (NLB) [45] and Closed Galaxy (CG) [46, 47, 48] models are variations of simple Leaky Box model. In these models, it is assumed that two types of confinement volumes are nested: the source regions and the Galaxy in the NLB model, and the galactic arm containing the Solar system and the extensive galactic halo in the CG model. However, there has been no direct evidence for such extra confinement volumes, and most cosmic-ray data are explained without such volumes except for sub-Fe/Fe ratio [33]. Even if the NLB model is adopted in order to fit the sub-Fe/Fe ratio, the results for $\bar{p}$ and $e^+$ hardly differ from those obtained by the SLB model because the production and interaction length of $\bar{p}$ and $e^+$ are much larger than those of Fe and sub-Fe. Thus we will not consider the NLB and CG models.
2.3 Models and the Present Calculations

From the above arguments, the simple Diffusion model was primarily studied in the following way. For stable particles (stable nuclei and antiprotons), we used the Leaky-Box approximation (SLB model), which is a simple mathematical approximation of the Diffusion model. In Chapter 3, we calculated $\bar{p}$ flux using the SLB model. We did not use the Leaky-Box approximation for $^{10}\text{Be}$ nuclei and positrons, which suffer intrinsic decay or synchrotron and inverse Compton energy losses everywhere in the Galaxy (not only in the galactic disk). In this case, we took a 1-D approximation (1-D Diffusion model), since the galactic disk is essentially uniform. In Chapter 5, we calculated positron flux using the 1-D Diffusion model. Finally, for calculations of $\bar{p}$'s from PBHs (Chapter 6), we took the 3-D Diffusion model because their sources, i.e., the evaporating PBHs, are assumed to be distributed spherically in the galactic halo.

In addition to the simple Diffusion model, we also studied the “Diffusive Reacceleration (DR) model,” [23, 24], where the second-order Fermi type of acceleration is considered to occur during propagation. Since such reacceleration probably occurs everywhere in the Galaxy, Leaky-Box approximation is not appropriate for this model. We used the 1-D halo model for all species, i.e., stable and radioactive nuclei (Chapter 4), antiprotons (Chapter 4), and positrons (Chapter 5).

2.4 Basic Equations for Diffusion and Leaky Box Models

Beginning with the 3-D Diffusion model, the general transport equation to treat spatial diffusion, energy loss and gain, and nuclear transformation, was first written by Ginzburg and Syrovatskii (for a review, see [10]),

$$\frac{\partial N_i}{\partial t} + \nabla(-D \nabla N_i + VN_i) + \left( \frac{1}{\tau_i} + \frac{1}{\gamma \tau_{\text{dec}}} \right) N_i + \frac{\partial}{\partial E_i} \left( \frac{dE_i}{dt} N_i \right) = Q_i + S_i, \quad (2.1)$$

where $N_i = N_i(E_i, \mathbf{r}, t)$ is the density of a given spices of cosmic-ray particle $i$ at position $\mathbf{r}$ with kinetic energy between $E_i$ and $E_i + dE_i$. As cosmic rays are almost isotropic, $N_i$ is related to the differential flux $I_i$, the number of particles per unit solid angle that pass per unit of time through a unit of area perpendicular to the direction of observation;

$$I_i = \frac{\beta c}{4\pi} N_i, \quad (2.2)$$

where $\beta c$ is the speed of the particle with $c$ being the speed of light. The term $-D \nabla N_i$ of eq. (2.1) represents the streaming of particles by diffusion, where $D = D(E_i, \mathbf{r})$ is the diffusion coefficient, while the term $VN_i$ represents the deterministic convection effect by the galactic wind with the velocity $\mathbf{V} = \mathbf{V}(\mathbf{r})$. With $dE_i/dt$, the mean rate at which a particle gains energy, the term $\partial/\partial E_i(dE_i/dt) N_i$ represents either energy loss (e.g., by ionization) or reacceleration of cosmic rays during propagation. The source term $Q_i = Q_i(E_i, \mathbf{r}, t)$ is the number of particles injected per
unit volume per unit time at position \( r \) and time \( t \) with the kinetic energy between \( E_{i} \) and \( E_{i} + dE_{i} \). The secondary source term \( S_{i} = S_{i}(E_{i}, r, t) \) gives the number of particles produced from other species or other energies by nuclear interactions. Particle loss through interaction with ISM and the intrinsic decay of radioactive nuclei are represented by the term \( \left( 1/\tau_{i} + 1/\gamma\tau_{dec} \right)N_{i} \), where \( \tau_{i} \) is the characteristic time of interaction, \( \tau_{dec} \) the life time of radioactive nuclei, and \( \gamma = (1 - \beta^{2})^{-1/2} \) the Lorentz factor.

We assume the galactic disk to be essentially uniform, so that all quantities are independent of the lateral distance parallel to the disk (1-D Diffusion model). In addition, if the diffusion coefficient \( D \) is assumed not to vary spatially and \( VN_{i} \) term is omitted based on the argument in section 2.2 (ignoring the Dynamical Halo model), then eq. (2.1) is reduced to

\[
\frac{\partial N_{i}}{\partial t} - D \frac{\partial^{2} N_{i}}{\partial z^{2}} + \left( \frac{1}{\tau_{i}} + \frac{1}{\gamma\tau_{dec}} \right) N_{i} + \frac{\partial}{\partial E_{i}} \left( \frac{dE}{dt} N_{i} \right) = Q_{i} + S_{i},
\]  

(2.3)

where \( z \) is the perpendicular distance from the galactic plane \( (z = 0) \). According to Nishimura et al. [43], the steady-state solution \( (\partial N_{i}/\partial t = 0) \) of eq. (2.3) with boundary condition

\[
N_{i}(z = \pm h_{\gamma}) = 0
\]

(2.4)
can be obtained as a Fourier series, whose details are described in Appendix C.

In the Leaky-Box approximation, the diffusion term in eq. (2.3) is replaced by the escape term,

\[
\frac{\partial N_{i}}{\partial t} + \left( \frac{1}{\tau_{esc}} + \frac{1}{\tau_{i}} + \frac{1}{\gamma\tau_{dec}} \right) N_{i} + \frac{\partial}{\partial E_{i}} \left( \frac{dE}{dx} N_{i} \right) = Q_{i} + S_{i}.
\]

(2.5)

Since interactions and ionization energy loss are both caused by ISM, it is convenient to introduce the traversed matter,

\[
x = \rho_{m}\beta ct,
\]

(2.6)

where \( \rho_{m} \) is the mean density of ISM. By inserting (2.6) and (2.2) into (2.3), we obtain

\[
\frac{\partial I_{i}}{\partial x} + \left( \frac{1}{\lambda_{esc}} + \frac{1}{\lambda_{i}} + \frac{1}{\rho_{m}\beta c\gamma\tau_{dec}} \right) I_{i} + \frac{\partial}{\partial E_{i}} \left( \frac{dE}{dx} I_{i} \right) = q_{i} + s_{i},
\]

(2.7)

where

\[
\lambda_{esc} = \rho_{m}\beta c\tau_{esc},
\]

(2.8)

\[
\lambda_{i} = \rho_{m}\beta c\tau_{int},
\]

(2.9)

\[
q_{i} = \frac{Q_{i}}{4\pi\rho_{m}},
\]

(2.10)

and

\[
s_{i} = \frac{S_{i}}{4\pi\rho_{m}}.
\]

(2.11)
Chapter 2. Cosmic-Ray Propagation Models

The general form of the secondary source term (2.11) is

\[ s_i(E_i) = \frac{1}{\rho_m} \sum_j^{CR} \int dE_j \left[ I_j(E_j) \sum_T^{ISM} \frac{d\sigma_{jT-i}}{dE_i} n_T \right], \]  

(2.12)

where \( d\sigma_{jT-i}/dE_i \) is the differential cross section of \( i \) particle production with kinetic energy \( E_i \) from the interaction of cosmic-ray particle \( j \) (flux \( I_j \)) and ISM target \( T \) (number density \( n_T \)). As ISM is mainly (\( \sim 90\% \)) composed of hydrogen atoms, the above term is often written as

\[ s_i(E_i) = \frac{1}{m_p^{CR}} \sum_j^{CR} \int dE_j \left[ I_j(E_j) \xi_{j-i} \frac{d\sigma_{jp-i}}{dE_i} \right], \]  

(2.13)

where \( m_p \) is the proton mass and \( d\sigma_{jp-i}/dE_i \) the differential cross sections for hydrogen target. The dimensionless factor \( \xi_{j-i} \) was introduced as a ratio of interaction cross section for mixed ISM to that of the pure hydrogen target, i.e.,

\[ \xi_{j-i}(E_j, E_i) \equiv \left( \sum_T^{ISM} \frac{d\sigma_{jT-i}}{dE_i} \left( \frac{d\sigma_{jp-i}}{dE_i} \right)^{-1} n_T \right) \left( \sum_T^{ISM} m_T n_T \right)^{-1}, \]  

(2.14)

where \( m_T \) is the mass of the target \( T \), and the relation

\[ \rho_m = \sum_T^{ISM} n_T m_T \]  

(2.15)

was used. In the same way, the interaction length \( \lambda_i \) in eq. (2.7) is expressed as

\[ \frac{1}{\lambda_i} = \left( \sum_T^{ISM} \sigma_{iT} n_T \right) \left( \sum_T^{ISM} m_T n_T \right)^{-1} = \frac{\xi_i \sigma_{ip}}{m_p}, \]  

(2.16)

\[ \xi_i(E_i) \equiv \left( \sum_T^{ISM} \frac{\sigma_{iT}}{\sigma_{ip}} n_T \right) \left( \sum_T^{ISM} m_T n_T \right)^{-1}, \]  

(2.17)

where \( \sigma_{ip} \) and \( \sigma_{iT} \) are the total interaction cross sections of cosmic-ray particle \( i \) with ISM hydrogen and \( T \) (including hydrogen) targets respectively.

If reacceleration is not considered (\( dE/dx < 0 \)), the solution of eq. (2.7) can be written as (see [10] [49])

\[ I_i(E_i) = \left( \frac{dE}{dx} \right)^{-1} \int_{E_i}^{\infty} dE_i' \left[ q_i(E_i') + s_i(E_i') \right] \]

\[ \times \exp \left[ \int_{E_i}^{E_i'} \frac{dE_i''}{dx} \left( \frac{1}{\lambda_{esc}(E_i'')} + \frac{1}{\lambda_i(E_i) + \frac{1}{\rho_m 3^\gamma \gamma''^{\tau_{dec}}}} \right) \right]. \]  

(2.18)
2.5 Solution for the Nuclei Component

For the propagation calculation of the nuclei component, it is convenient to use kinetic energy per nucleon \( E_i^A = E_i/A_i \) (\( A_i \) the mass number of \( i \) particle) instead of kinetic energy, because \( E^A \) remains essentially unchanged in the spallation process, i.e.,

\[
\frac{d\sigma_{JT-i}}{dE_i^A} = \sigma_{JT-i}(E_i^A)\delta(E_j^A - E_i^A). \tag{2.19}
\]

In this case, eq. (2.7), eq. (2.13), eq. (2.14), and eq. (2.18) become

\[
\frac{\partial I_i}{\partial x} + \left( \frac{1}{\lambda_{esc}} + \frac{1}{\lambda_i} + \frac{1}{\rho_m\beta_c\gamma\tau_{dec}} \right) I_i + \frac{\partial}{\partial E_i^A} \left( \frac{dE_i^A}{dx} I_i \right) = q_i + \frac{1}{m_p} \sum_{j>i}^{CR} I_j \xi_{j-i} \sigma_{jp-i},
\]

\[
I_i(E_i^A) = \left( \frac{dE_i^A}{dx} \right)^{-1} \int_{E_i^A}^{E_i^A} dE_i^A' \left[ q_i(E_i^A') + \frac{1}{m_p} \sum_{j>i}^{CR} I_j(E_i^A') \xi_{j-i}(E_i^A') \sigma_{jp-i}(E_i^A') \right]
\times \exp \left[ \int_{E_i^A}^{E_i^A'} dE_i^A'' \left( \frac{dE_i^A''}{dx} \right)^{-1} \left( \frac{1}{\lambda_{esc}(E_i^A'')} + \frac{1}{\lambda_i(E_i^A'')} + \frac{1}{\rho_m\beta''c\gamma''\tau_{dec}} \right) \right].
\tag{2.22}
\]

Since the secondary source is contributions from heavier elements only \((j>i)\), the solution for the heaviest nuclei can be obtained first, followed by that for the next to heaviest nuclei, and so on. If the nuclei are stable \((\tau_{dec} = \infty)\), the only input parameters are the primary sources \(q_i\) and escape length \(\lambda_{esc}\). If the nuclei are radioactive, \(\rho_m\) is added. This is the reason that radioactive nuclei are sensitive to the confinement time \(\tau_{esc}\) and halo thickness \(h_h\). The relation among \(\rho_m\), \(\tau_{esc}\), and \(h_h\) will be explained later. To determine the above parameters, calculations of nuclei fluxes were performed to fit the observational data as follows.

2.6 Calculation of Nuclei Flux and Parameterization of Escape Length

Since light \((^3\text{Li}, ^4\text{Be}, ^5\text{B})\) nuclei are almost absent as end products of stellar nucleosynthesis, the relatively abundant those elements in cosmic rays should be secondary products from the interaction (spallation) of medium \((^6\text{C}, ^7\text{N}, ^8\text{O})\) or heavier nuclei with ISM. The “secondary” \((\text{Li, Be, B})\) to “primary” \((\text{C, N, O})\) ratio is therefore the most sensitive observable to the amount of the interstellar matter that is traversed by cosmic rays, represented by the “escape length” \(\lambda_{esc}\). The boron to carbon \((\text{B/C})\) ratio is most frequently used to obtain \(\lambda_{esc}\) because boron is most abundant among the secondaries and mainly produced from carbon. Actually 60
to 70 % of boron is produced from carbon under the SLB model. Oxygen (O) also produces \( \sim 20 \% \) of boron. The elements from neon \((\text{Ne})\) to silicon \((\text{Si})\) contribute \( \sim 7 \% \) of the boron production, while that from elements heavier than silicon accounts for less than 1 %. In the following calculation, we took elements lighter than silicon (including silicon) into account, although only fluxes of the most important elements, i.e., B, C, and O were shown in the figures.

For observational data, we used (as shown in Fig. 2.1, etc.) the high-energy data of HEAO-3-C2 [32], Gupta et al. [54], the low-energy data of Garcia-Munoz et al. [33], ISEE-3 experiment [38, 55], and Voyager experiment [35, 56]. For cross sections of nuclear spallation induced by hydrogen target \((\sigma_{ijp} \text{ and } \sigma_{ijp-i})\), we utilized the calculation program written by Webber et al. [50], which is based on the data of their systematic measurements using 42 beams of 12 separate nuclei from \(^{12}\text{C}\) to \(^{58}\text{Ni}\) [51]. This program can predict the cross sections of the hydrogen target and beam nuclei with \(Z = 4 - 28, A = 7 - 60\) above \(\sim 200 \text{ MeV/nucleon}\) to an accuracy \(\sim 10 \%\) or better [51]. Per ref. [32], we took the ISM composition of 90 % hydrogen and 10 % helium atoms \((n_p = 0.9 \text{ and } n_{\text{He}} = 0.1)\). As for the spallation cross sections of the helium target \((\sigma_{i\text{He}} \text{ and } \sigma_{j\text{He}-i})\), we utilized the empirical formula of Ferrando et al. [52] based on their measurements using carbon, oxygen, and iron beams.

Ionization loss is the only energy changing process. Energy loss rate \((-dE/dx)\) by ionization of 90 % hydrogen and 10 % helium ISM is calculated using the Bethe-Bloch equation. In addition, we took ionized hydrogen in ISM into account according to Soutoul et al. [53] (see also [35]).

With those inputs and eq. (2.22), we perform the calculation of nuclei flux and tuning of the escape length and source spectra as follows. First, we use the relative source abundances obtained by Engelmann et al. [32], and then we will tune them if the calculated fluxes do not fit the data, although we expect that such tuning is not necessary because we use the same observational data and nuclear spallation cross sections as Engelmann et al.

As expected, it is shown that the source abundances of C and O given in ref. [32] reproduce the observations by comparing the calculated O flux and C/O flux ratio with the data (Fig. 2.2). The energy spectra at sources are tuned to fit the absolute flux of primaries. We use oxygen flux (Fig. 2.2(d)) for this fitting, since more than 90 % of oxygen is primary. The energy spectrum of oxygen flux is, however, slightly dependent also on \(\lambda_{\text{esc}}\), which is mainly determined to fit B/C and C/O flux ratios. Then, to obtain the self-consistent source spectra and escape length, the energy-dependent \(\lambda_{\text{esc}}\) is initially tuned to fit B/C and C/O ratios, after which the source spectra are tuned to fit the oxygen absolute flux. Since the B/C and C/O ratios are slightly changed by source spectra tuning, \(\lambda_{\text{esc}}\) is again tuned to fit the B/C and C/O ratios. This procedure is repeated until the close fit is obtained.

Fig. 2.1 and 2.2 show the results of our fitting. The obtained source spectrum is

\[
g_i dE^A = a_i (P/A_i)^{-2.3} \beta^{-1} dE^A, \tag{2.23}
\]

where \(P\) is momentum of the particle and \(a_i\) the relative abundances of ref. [32]. The source spectrum (eq. (2.23)) is close to that obtained in [32].
Figure 2.1: Cosmic-ray nuclei flux: calculations using SLB model and observations. Solar modulation calculations are for the observation period of HEAO-3 experiment \[32\] (\(\phi = 600 \text{ MV}\)), Garcia-Munoz et al. \[33\] (\(\phi = 490 \text{ MV}\)), ISEE-3 experiment \((^{10}\text{Be}/^{9}\text{Be})\) \[55\] (\(\phi = 650 \text{ MV}\)), and Voyager experiment \((^{10}\text{Be}/^{9}\text{Be})\) \[56\] (\(\phi = 500 \text{ MV}\)). Details of solar modulation are described in Appendix B. (a) Cosmic-ray boron to carbon flux ratio. Data points are from the HEAO-3 experiment \[32\] (filled circles), Garcia-Munoz et al. \[33\] (filled squares), Gupta et al. \[54\] (open triangles), ISEE-3 experiment \[38\] (open circles), Voyager experiment \[35\] (open square), and compilation by Ormes et al. \[57\] (error bars). (continued to next page)
Figure 2.2: (Continued) (b) Cosmic-ray radioactive beryllium ($^{10}$Be) to stable beryllium ($^{9}$Be) flux ratio shown for the three values of $n_{ISM}$, the mean number density of ISM. Data points are from the ISEE-3 experiment [55] (filled triangle), and Voyager experiment [56] (open square). (c) Cosmic-ray carbon to oxygen flux ratio. Data points are from the HEAO-3 experiment [32]. (d) Cosmic-ray oxygen flux. Data points are from the HEAO-3 experiment [32].
Figure 2.3: Escape length for nuclei with $A/Z = 2$. The thick solid and dashed lines respectively are the present parameterizations indicated by eqs. (2.24) - (2.28) and eqs. (2.29) and (2.30). The thin lines are from Engelmann et al. [32], Webber et al. [2], Stephens et al. [3], and Simon et al. [4, 24].
Figure 2.4: Same as Fig. 2.3 for protons and antiprotons. $\lambda_{esc}(R, \beta)$ (thick solid line) and $\lambda_{esc}(R)$ (thick dashed line) do not agree at low energies. The previous works (thin lines) also show discrepancies at low energies.
The resultant escape length is parameterized as (also shown in Fig. 2.3)

\[ \lambda_{esc} = \lambda_{esc}(R, \beta) = \lambda_1(R)\lambda_2(\beta), \] (2.24)

\[ \lambda_1(R) = \begin{cases} 17.0 & (R < 4.4GV) \\ 17.0 \times (R/4.4GV)^{-0.65} & (R > 4.4GV), \end{cases} \] (2.25)

\[ \lambda_2(\beta) = \begin{cases} (\beta/0.9)^3 & (\beta < 0.9) \\ 1 & (\beta > 0.9), \end{cases} \] (2.27)

where \( R \) is the rigidity of particle.

As previously shown \([32, 33, 34, 35, 36, 37, 38, 39]\), the escape length peaks at the kinetic energy around 1.5 GeV/n. The fall of high-energy side fits the power of rigidity (eq. (2.26)) and can be understood through the power-law spectrum of interstellar magnetic irregularity. On the other hand, the fall of low-energy side is not well understood. In the above parameterization, it is assumed that this fall is related to the speed of the particle, i.e., \( \lambda_{esc} \propto \beta^3 \) being in good agreement with the results of Ormes and Protheroe \([57]\). Although the possibility that the escape length is related to \( \beta \) arises from the diffusion-convection effect in the Dynamical Halo model \([44]\), Ormes and Protheroe have shown that the fall of the escape length is much steeper than expected in the Dynamical Halo model \([57]\). In addition, as already stated, the convection velocity is stringently limited \([40]\) by the nuclei data. Therefore the origin of this fall in escape length is unknown at present and it is just an assumption that the fall is related to \( \beta \).

Ormes and Protheroe \([57]\) also proposed the function form of \( \lambda_{esc} \) which is dependent only on rigidity. By using this form, we obtained another parameterization of \( \lambda_{esc} \), i.e.,

\[ \lambda_{esc} = \begin{cases} 17.0 \times \left\{ 1 + \frac{(R_0/R)^2}{1 + (R_0/4.4GV)^2} \right\}^{-3/2} & (R < 4.4GV) \\ 17.0 \times (R/4.4GV)^{-0.65} & (R > 4.4GV), \end{cases} \] (2.29)

where \( R_0 = 2m_pc/e = 1.88 \) GV. This escape length also well fit the B/C ratio as shown in Fig. 2.5 by dashed lines. In this parameterization, the escape length is assumed to be a function of rigidity only. One can speculate on such dependence if the falls in escape length at both high- and low-energy sides are due to the spectrum of magnetic irregularity. Although there exist other possibilities on the functional forms of \( \lambda_{esc} \), the above two types of parameterization have the firmest basis because rigidity and speed are most important parameters for particles that propagate in turbulent magnetic field. Therefore, we will study these two parameterizations only.

For the nuclei with \( A/Z = 2 \), the two parameterizations, i.e., \( \lambda_{esc}(R, \beta) \) (eqs. (2.24) - (2.28)) and \( \lambda_{esc}(R) \) (eqs. (2.29) and (2.30)) agree as shown in Fig. 2.3 because both are obtained to fit the fluxes of light and medium nuclei, most of which satisfy \( A/Z \approx 2 \). However, the two parameterizations do not agree for antiprotons as shown in Fig. 2.4 because their mass to charge ratio is \( A/Z = 1 \), which is different from that of nuclei. Previous works (thin lines, Fig. 2.4) also show discrepancies at low
energies because both functional forms exist in the previous works, i.e., the parameterization by Engelmann et al. [32] and that by Webber et al. [2] are $\lambda_{\text{esc}}(R, \beta)$, while that by Simon et al. [4, 24] is $\lambda_{\text{esc}}(R)$.

In this way, the escape length of $\bar{p}$ contains an ambiguity concerning the functional forms of the escape length. For example, at the kinetic energy of 1 GeV, $\lambda_{\text{esc}}(R)$ is smaller than $\lambda_{\text{esc}}(R, \beta)$ by a factor of 2.3 resulting in ~2 times smaller $\bar{p}$ flux as shown in the next chapter (Fig. 3.10). Antiprotons having an energy of 1 GeV in interstellar space are observed at 1 AU (at the top of the atmosphere) in the energy range between 100 and 500 MeV due to the solar modulation effect (Appendix B). Then such low-energy $\bar{p}$ flux at 1 AU is expected to differ by a factor of ~2 when using different parameterizations of $\lambda_{\text{esc}}$. Therefore, in order to calculate low-energy $\bar{p}$ flux to compare it with the recent observations, we must determine which parameterization is correct phenomenologically.

To do this, positron flux was utilized (see Chapter 5). Positrons are considered to be mainly secondaries from the comparison between calculated and observed high-energy (above several GeV) fluxes. If so, their source spectrum at low energies is also calculable. Then, by comparing the source spectrum and the spectrum of observed flux, we can gain data on positron escape length. Fig. 2.6 shows the two parameterizations of $\lambda_{\text{esc}}$ for electron and positrons ($e^{\pm}$). Because of the low mass (511 keV), $\lambda_{\text{esc}}(R, \beta)$ for $e^{\pm}$ does not fall off even at energies as low as 100 MeV, while $\lambda_{\text{esc}}(R)$ falls off similarly to the case of nuclei, protons and antiprotons. At an energy of 1 GeV, $\lambda_{\text{esc}}(R)$ of $e^{\pm}$ is smaller than $\lambda_{\text{esc}}(R, \beta)$ by a factor of as large as 7. Like $\bar{p}$, these differences result in differences of low energy (100 to 500 MeV)
Figure 2.6: Same as Fig. 2.3 for electrons and positrons.

electron and positron flux by a factor of $\sim 5$ to 10. When considering the fact that the flux of electrons plus positrons is measured with relatively high precision [85, 86, 87], and that the secondary positron source spectrum is determined only by the parent proton flux and positron production cross sections, the differences in $\lambda_{esc}$ are expected to result in that in positron fraction $e^+/(e^- + e^+)$. Since there is some observational data of this low-energy region (Fig. 1.2), we can determine which parameterization better fits the data and should be taken. We will perform such analyses in Chapter 5.

2.7 Differences between SLB and Diffusion Models

In this section, the results of nuclei flux calculations using the 1-D Diffusion model are shown and compared with the SLB results. As already stated, the Leaky-Box approximation is expected to be valid for stable nuclei because they traverse galactic plane many times (and will be observed many times) before occurring interactions and escape.

For the calculation of the 1-D Diffusion model, we utilized the Fourier series solution (for example, see Nishimura et al. [43], see also Appendix C). The half thicknesses of the source region and ISM distribution, and the ISM density at the galactic plane were fixed to $h_s = h_g = 100$ pc, and $\rho_0 = 1.1$ atoms cm$^{-3}$ respectively. The half thickness of the halo $h_h$ was determined to fit the $^{10}$Be data as shown later. Fig. 2.7 and 2.8 show the results of the nuclei flux calculation using the 1-D Diffusion model with $\lambda_{esc}(R, \beta)$ (eqs. (2.24) - (2.28)). As expected, the results for stable nuclei (O, C, B) are in good agreement with those obtained with the SLB model (Fig. 2.1
and Fig. 2.2). This fact assures us that the calculation of $\bar{p}$ flux can also be carried out in the SLB model without obtaining conflicting results with the Diffusion model because the interaction length of $\bar{p}$ is longer than that of light and medium nuclei.

On the other hand, the results for $^{10}\text{Be}$ are different between SLB and Diffusion models. Fig. 2.1 (b) shows the $^{10}\text{Be}/^9\text{Be}$ flux ratio calculated using the SLB model with the mean ISM density $n_{\text{ISM}} \equiv \sum n_T = 0.4$, 0.3, and 0.2 atoms cm$^{-3}$ from above. This flux ratio is essentially not dependent on the escape length $\lambda_{\text{esc}}$ because both $^{10}\text{Be}$ and $^9\text{Be}$ are secondaries produced from the same parents (C, O, and small contribution from heavier nuclei). The ratio is dependent on the confinement time $\tau_{\text{esc}}$ only because only $^{10}\text{Be}$ nuclei decay while $^9\text{Be}$ nuclei do not decay. From eq. (2.8), the confinement time is written as

$$\tau_{\text{esc}} = \frac{\lambda_{\text{esc}}}{n_{\text{ISM}} m_{\text{ISM}} \beta c},$$

(2.31)

where $m_{\text{ISM}} = \rho_m/n_{\text{ISM}}$ is the mean mass of the ISM atoms (see also eq. (2.15)). Since $\lambda_{\text{esc}}$ is determined by $B/C$ and other flux ratios much more precisely than $\tau_{\text{esc}}$, $\tau_{\text{esc}}$ and $n_{\text{ISM}}$ are almost in a one-to-one correspondence from the above equation. Therefore, $^{10}\text{Be}/^9\text{Be}$ flux ratio seems to be dependent on $n_{\text{ISM}}$.

Fig. 2.7 (b) shows the same flux ratio calculated using the 1-D Diffusion model with $h_h = 1, 2, \text{and } 4 \text{kpc}$. The halo thickness in the Diffusion model is related to the mean ISM density in the SLB model [41], namely, the large halo corresponds to the small ISM density and the small halo the large ISM density, because the ISM density at the galactic plane $\rho_0$ and the thickness of the ISM distribution $h_g$ are fixed. However, this relation is dependent on energy. As shown in Fig. 2.9, the energy dependences of the $^{10}\text{Be}/^9\text{Be}$ flux ratio are different between the Diffusion and SLB models. This energy dependence arises from the Lorentz factor of decaying nuclei, i.e., the life time of high-energy $^{10}\text{Be}$ is longer than that of the low-energy one due to the Lorentz delay, resulting in small loss of $^{10}\text{Be}$ by decay at high energy, then $^{10}\text{Be}/^9\text{Be}$ flux ratio increases with energy. Finally, the flux ratio reaches an asymptotic value at a high energy limit, where $^{10}\text{Be}$ hardly decays during the confinement time. The asymptotic value of the flux ratio is thus roughly equal to the production ratio of $^{10}\text{Be}$ to $^9\text{Be}$. When considering this origin of the energy dependence of $^{10}\text{Be}/^9\text{Be}$ flux ratio, the fact that energy dependence differs between the Diffusion and SLB models means that the relation between $h_h$ and $n_{\text{ISM}}$ is dependent on the life time of the particle.

This nature was pointed out by Ptuskin et al. [41], who discussed the surviving fractions of radioactive nuclei with different life times, i.e., $^{10}\text{Be}$ and $^{26}\text{Al}$. $^{10}\text{Be}$ with different energy shows the same nature. At a given $h_h$, for long life time particles, $n_{\text{ISM}}$ seems to be small, because they diffuse far from the gaseous disk, while for the short life time particles, the same $h_h$ corresponds to a larger $n_{\text{ISM}}$, because they decay around the disk before diffusing into high $z$. Therefore, it is of great importance to observe energy dependence of the $^{10}\text{Be}/^9\text{Be}$ flux ratio in order to confirm the diffusion picture of the cosmic-ray propagation. Although nobody has observed it, here we consider the results of 1-D Diffusion model (solid lines in Fig. 2.9) only because (i) 1-D Diffusion model is based on a realistic picture of the cosmic-ray
Figure 2.7: Cosmic-ray nuclei flux: Calculations using the Diffusion model and observations. Data points and solar modulation calculations are the same as Fig. 2.1 and 2.2. (b) Shown for the three values of the half thickness of halo $h_h = 1$, 2, and 4 kpc.
Figure 2.8: (Continued)
Figure 2.9: \(^{10}\text{Be}/^{9}\text{Be}\) ratio in the Diffusion and SLB models. Data points and solar modulation calculations are the same as Fig. 2.1 and 2.2. The solid lines are the results of the Diffusion model with \(h_h = 1, 2, 4\) kpc from above. The dashed lines are the results of the SLB model with \(n_{ISM} = 0.4, 0.3,\) and \(0.2\) atoms cm\(^{-3}\) from above.

propagation, and (ii) the SLB model is its simple mathematical approximation. In other words, we have shown by Fig. 2.9 that the SLB approximation is not valid for radioactive nuclei. If so, the electron/positron flux also should be calculated using at least the 1-D Diffusion model because the nature of electron/positron interaction is also closely related to the size of the halo [43].

Based on the above arguments, we use SLB model for \(\bar{p}\) flux calculation (Chapter 3) and use 1-D Diffusion model with \(h_h = 1, 2,\) and \(4\) kpc for \(e^+\) flux calculation (Chapter 5).
Chapter 3

Calculation of Cosmic-Ray Antiproton Flux using the Standard Leaky Box Model

The calculation of $\bar{p}$ flux under the SLB model requires the following inputs (see also eq. (3.12)).

<table>
<thead>
<tr>
<th>Input</th>
<th>Reference</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interstellar proton flux $I_p$</td>
<td>LEAP [58]</td>
<td>section 3.1</td>
</tr>
<tr>
<td>$\bar{p}$ production cross sections $\sigma_{pp\rightarrow\bar{p}}$</td>
<td>Tan and Ng [65]</td>
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</tr>
<tr>
<td>Contribution of nuclei $\epsilon$</td>
<td>Gaisser et al. [1]</td>
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<td>Escape length $\lambda_{esc}$</td>
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<td>$\bar{p}$ interaction cross sections $\sigma_{pp}$</td>
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<td>section 3.5</td>
</tr>
<tr>
<td>Differential cross sections $d\sigma_{pp\rightarrow\bar{p}}/dE_{\bar{p}}$</td>
<td></td>
<td>section 3.6</td>
</tr>
</tbody>
</table>

The third column shows the references for input data (parameters) used in our calculation. Each input will be examined in the following sections. We used new parameterizations of the escape length obtained in the previous chapter. With this, ambiguity of the $\bar{p}$ flux associated to that of the escape length is considerably reduced. Further, we have developed a new parameterization of the differential cross sections of $\bar{p}p \rightarrow \bar{p}pX$ interactions (section 3.6). This interaction is particularly important for low-energy $\bar{p}$’s because it possibly produces a considerable amount of low-energy $\bar{p}$’s [66]. From our new parameterization, it was found that some of the previous authors [66, 2, 3] overestimated low-energy $\bar{p}$’s produced from $\bar{p}p$ non-annihilation interaction. Comparison with previous works are shown in section 3.7.
3.1 Interstellar Proton Flux

Choosing an appropriate interstellar (IS) proton flux is an important first ingredient in calculating $\bar{p}$ flux. As the IS proton flux cannot be measured directly at 1 AU (at the top of the atmosphere (TOA)) due to the solar modulation effect, it should be deduced using the models of solar modulation. Fortunately, the protons that effectively produce $\bar{p}$'s are those in the energy region above 10 GeV, where the solar modulation effect is small and the IS proton flux can essentially be determined by the observations at TOA.

In 1987 (near the last solar minimum as shown in Fig. 3.2), the LEAP (Low Energy Antiproton) balloon-flight experiment measured proton flux over a wide energy range from 200 MeV to 100 GeV with a single instrument [58] (solid circles, Fig. 3.1). By using the spherically symmetric diffusion-convection model (see Appendix B) to evaluate the solar modulation effect, proton flux measured by the LEAP experiment was fitted by the IS proton flux

$$I_p = 1.5 \times 10^4 \beta^{-1} P^{-2.74} \text{ (m}^{-2}\text{sr}^{-1}\text{s}^{-1}\text{GeV}^{-1}),$$

(3.1)

with the “modulation parameter” $\phi = 500$ MV, where $P$ is the momentum of protons. As shown in Appendix B, $\phi$ is the mean energy loss in the Solar system divided by the particle charge. The above IS flux and modulation parameter agree with analyses by the LEAP group [58]. Proton fluxes measured by the BESS experiment [12, 13] were also fitted by the same IS flux, eq. (3.1), with $\phi = 700$ and 600 MV for 1993 and 1994 flight respectively.

To confirm that the above variation in the $\phi$ parameter is consistent with the solar activity cycle, we utilized the data of the CLIMAX neutron monitor [59] provided by the University of Chicago [60]. Fig. 3.2 shows the time variation of the 27-day averaged count rate of the CLIMAX neutron monitor from 1953 to 1995. The neutron monitor detects, at large atmospheric depths, variations of intensity in the cosmic-ray spectrum at TOA. Interactions of the cosmic rays with the atmosphere produce, among other things, a lower energy nucleonic component consisting of nucleons, in particular neutrons that are not slowed by ionization loss. These secondaries fall in the energy range of a few hundred MeV up to about one GeV. Because of the falling energy spectrum of the cosmic rays at TOA, the neutron monitors are most sensitive to the low energy (1-20 GeV) portion of the TOA spectrum, which is most affected by the solar modulation effect. As shown in Fig. 3.2, the count rate of the CLIMAX neutron monitor shows 11- and 22-yr solar cycles. The flight dates of LEAP and BESS experiments indicated by arrows are near the solar minimum, which is consistent with the small $\phi$ values obtained by the proton flux fitting.

This correlation between the neutron monitor count rate and $\phi$ parameter was clearly shown by Evenson et al. [61] (Fig. 3.3), who used the spherically symmetric diffusion-convection model, the same model as we use here. As shown in this figure, the modulation parameter obtained by the proton spectra measured by LEAP and BESS experiments are in good agreement with the relation shown by Evenson et al.
Chapter 3. Calculation of Cosmic-Ray Antiproton Flux using the SLB Model

Figure 3.1: Proton flux at TOA around the solar minimum. Data points are from LEAP experiment [58], IMP-8 experiment (reproduced from [58]), and BESS experiment [13]. Date of the flight is indicated in Fig. 3.2. Lines are the calculated flux from the IS proton flux of eq. (3.1) with the solar modulation effect included using the spherically symmetric diffusion-convection model.
Figure 3.2: Variation of 27-day averaged count rate of the CLIMAX neutron monitor [60] from 1953 to 1995 (upper), and a magnified view (lower) from 1993 to 1995. The horizontal axes represent the date as, e.g., 94.0 corresponds to January 1, 1994. Flight date of LEAP and BESS experiments are indicated by arrows.
Figure 3.3: Relation between the modulation parameter and count rate of the CLIMAX neutron monitor. Circles are from Evenson et al. [61] (reproduced from [33]). Solid and open stars are for LEAP and BESS experiments. Modulation parameters for these experiments are obtained by fitting the proton spectra (see text).
Figure 3.4: IS proton flux of Webber et al. [2, 124] (dashed line), compared with our fitting based on LEAP and BESS data (solid line).

From the above analyses, it was found that solar modulation was correctly evaluated and that the IS proton flux indicated by eq. (3.1) is consistent with recent observations by the LEAP and BESS experiments. Therefore, we used eq. (3.1) in the following calculations. As shown in Fig. 3.4, however, Webber et al. [2, 124] reported the IS proton flux to be larger by a factor of 1.6. This discrepancy is mainly due to a discrepancy in the observed fluxes at TOA [62]. Although we cannot discuss this discrepancy in the observations, we will show that this is the most serious problem in calculating the expected secondary $\bar{p}$ flux. More precise measurement in the near future (e.g., by the BESS experiment) should reconcile this discrepancy.

Cosmic rays heavier than protons can also produce antiprotons, which will be discussed in section 3.3. The uncertainty in the heavy primary contributions is not so serious problem as the proton flux uncertainty because (i) contributions from heavy cosmic rays are relatively small ($\sim 20\%$) and (ii) the flux ratios of heavy cosmic rays to protons are known more precisely than the absolute flux of protons.

### 3.2 Antiproton Source Spectrum

When high-energy cosmic-ray nuclei interact with ISM, nucleon-antinucleon pairs are created when the available energy in the center of momentum system (CMS) exceeds the mass of this pair. In interstellar space, the dominant mode of $\bar{p}$ production is $pp \rightarrow \bar{p}X$ and $pp \rightarrow \bar{n}X$, where $\bar{n}$ subsequently decays into $\bar{p}$. Hereafter, cross sections of $pp \rightarrow \bar{n}X$ interactions are assumed to be equal to those of $pp \rightarrow \bar{p}X$ interactions with the momenta of outgoing $\bar{p}$'s equal to those of $\bar{n}$'s. This relation
is induced from the “isospin symmetry” of hadron interactions. The kinetic energy threshold of $pp \rightarrow \bar{p}X$ is $E_{th} = 5.6$ GeV in the laboratory system (LS). Measurements of the cross sections of this interaction $\sigma_{pp \rightarrow \bar{p}}$ have been carried out over a kinetic energy range from 12 to 400 GeV [63]. These data are adequate to calculate the cosmic-ray $\bar{p}$ production rate. Although no measurement has been carried out for the proton energy between $E_{th}$ and 12 GeV, contributions from those protons near the threshold are very small and the ambiguity of the cross sections of this energy range is not a problem.

Stephens [64] developed an analytical representation for the Lorentz-invariant differential cross section of $pp \rightarrow \bar{p}X$ interaction $E d^3 \sigma_{pp \rightarrow \bar{p}}/d\bar{p}^3$. Independently, Tan and Ng [65] used a different set of parameters. These invariant cross sections are easily converted to differential cross sections in LS $d\sigma_{pp \rightarrow \bar{p}}/dE_{\bar{p}}$ (Fig. 3.5), where $E_{\bar{p}}$ is the kinetic energy of produced $\bar{p}$’s. As shown in Fig. 3.5, two parameterizations agree within a factor of 1.2. Here we take the parameterization of Tan and Ng, but this is not a critical choice.

This figure also shows an important feature of antiproton production – the energy spectra of the produced $\bar{p}$’s have peaks at 1 to several GeV with both high- and low-energy $\bar{p}$ productions being strongly suppressed. The reason is that (i) $\bar{p}$’s at rest in CMS are most frequently produced while $\bar{p}$’s having high momenta are rarely produced, and (ii) CMS is boosted forward in LS because of the high energy of incident cosmic-ray protons, so that the $\bar{p}$’s at rest in CMS correspond to 1 to several GeV in LS. Actually, $\bar{p}$’s at rest in CMS correspond to $E_{\bar{p}} = 1.4, 2.9, 6.0, \text{ and } 11.0$ GeV for incident proton energies $E_p = 10, 30, 100, \text{ and } 300$ GeV respectively. The expected suppression of low-energy cosmic-ray $\bar{p}$’s originates from these kinematical features.

By convolving the IS proton flux ($I_p$) of eq. (3.1) with $\sigma_{pp \rightarrow \bar{p}}$ of Tan and Ng, we obtained a spectrum of the production rate of $\bar{p}$’s in interstellar space (“$\bar{p}$ source spectrum” shown in Fig. 3.6 by thick solid line). Note that this source spectrum does not include the contributions from heavier nuclei, which will be considered in the next section. Also shown in the figure are the source spectra calculated using other combinations of $I_p$ and $\sigma_{pp \rightarrow \bar{p}}$, i.e., $I_p$ of eq. (3.1) and $\sigma_{pp \rightarrow \bar{p}}$ of Stephens (thick dashed line), and $I_p$ of Webber et al. (dashed line of Fig. 3.4) and $\sigma_{pp \rightarrow \bar{p}}$ of Stephens (thick dotted line), which are larger than our original spectrum shown by the thick solid line by a factor of 1.2 and 1.9 respectively at the peak energy of 2 GeV. In addition, the source spectrum reported by Webber et al. [2] is also shown by thin dash-dotted line. They obtained this spectrum using the $I_p$ of Webber et al. and $\sigma_{pp \rightarrow \bar{p}}$ of Stephens, which agree with our calculation using the same combination (thick dotted line). From these comparison, we found that the ambiguity of the $\bar{p}$ source spectrum is mainly due to that of proton flux, while the ambiguity due to the $\bar{p}$ production cross sections is relatively small.
Figure 3.5: Differential cross sections of $\bar{p}$ production in $pp$ collisions for the incident $p$ energy $E_p = 10, 30, 100,$ and $300$ GeV. Solid and dashed lines are the parameterizations by Tan and Ng [65] and Stephens [64], respectively.
Figure 3.6: Interstellar \( \bar{p} \) production rate in \( pp \) collisions calculated using various combinations of interstellar proton flux \( I_p \) and \( \bar{p} \) production cross sections \( \sigma_{pp\rightarrow\bar{p}} \) (thick lines, see text). Thin dot-dashed line is from Webber et al. [2], who obtained it using the same combination as that of the thick dotted line.
3.3 Contributions from Heavy Cosmic Rays and Targets

Antiproton production from heavy cosmic rays and heavy targets was carefully estimated by Gaisser et al. [1]. Since the antiprotons are produced from cosmic-ray nuclei as well as protons, the source term for antiprotons is (see also eq. (2.13))

$$s_p(E_{\bar{p}}) = \frac{2}{m_p} \sum_j^{CR} \int_{E_{th}}^{\infty} dE_j \left[ I_j(E_j^A) \xi_{j-p} \frac{d\sigma_{jp-\bar{p}}}{dE_{\bar{p}}} \right],$$

where the factor of 2 accounts for \( \bar{p} \) production by \( \bar{n} \) decay. The factor for mixed ISM (called “ISM factor”, see also eq. (2.14)) is

$$\xi_{j-\bar{p}} = \left\{ \frac{\sum_j^{ISM} \frac{d\sigma_{jT-\bar{p}}}{dE_{\bar{p}}} \left( \frac{d\sigma_{jp-\bar{p}}}{dE_{\bar{p}}} \right)^{-1}}{n_T} \right\} \left( \frac{\sum_j^{ISM} m_T n_T}{m_p n_T} \right)^{-1}.$$  \(3.3\)

Although the \( \bar{p} \) production from \( p \) nucleus collisions is essentially explained as coming from \( p \) nucleon collisions, the Fermi momentum of the target nuclei affects the kinematics of \( \bar{p} \) production (Appendix A). Thus the differential cross sections for the heavy targets \( d\sigma_{jT-\bar{p}}/dE_{\bar{p}} \) should have different energy (both \( E_j \) and \( E_{\bar{p}} \)) dependence from that of proton targets \( d\sigma_{jp-\bar{p}}/dE_{\bar{p}} \). However, the production spectra of \( \bar{p} \)'s above \( \sim 700 \text{ MeV} \) are not affected very much by Fermi momentum. When considering this feature along with the fact that it is difficult to observe \( \bar{p} \)'s having energies less than \( \sim 700 \text{ MeV} \) in interstellar space at 1 AU due to the adiabatic energy loss by the solar wind (see Fig. B.1, Appendix B), it is valid that the energy dependences of \( d\sigma_{jT-\bar{p}}/dE_{\bar{p}} \) are taken to be the same as that of \( d\sigma_{jp-\bar{p}}/dE_{\bar{p}} \). Then, \( \xi_{j-\bar{p}} \) in eq. (3.3) is written as it is independent of \( E_j \) and \( E_{\bar{p}} \).

As for the contributions from heavy cosmic rays, we introduce the approximations as follows [1]. First, the energy spectra of heavy cosmic rays are taken to be the same as that of protons,

$$I_j(E_j^A) = r_j I_p(E_p),$$

where \( r_j \) is the relative abundances of each nuclei component to proton. Further, the energy (both \( E_j \) and \( E_{\bar{p}} \)) dependence of \( \bar{p} \) production cross sections for the cosmic rays \( d\sigma_{jp-\bar{p}}/dE_{\bar{p}} \) is taken to be the same as that for protons \( d\sigma_{jp-\bar{p}}/dE_{\bar{p}} \). This approximation is also valid for the same reason that the same approximation for heavy targets is valid (see above). With those approximations, the source term is reduced to [1]

$$s_p(E_{\bar{p}}) = \frac{2\epsilon}{m_p} \int_{E_{th}}^{\infty} I_p(E_p) \frac{d\sigma_{pp-\bar{p}}}{dE_{\bar{p}}} dE_{\bar{p}},$$

where

$$\epsilon = \sum_j^{CR} \left[ r_j \xi_{j-\bar{p}} \frac{d\sigma_{jp-\bar{p}}}{dE_{\bar{p}}} \left( \frac{d\sigma_{pp-\bar{p}}}{dE_{\bar{p}}} \right)^{-1} \right].$$

(3.6)
By inserting eq. (3.3) into eq. (3.6), we obtain
\[ \epsilon = \left( \sum_T \epsilon_T n_T \right) \left( \frac{m_T}{m_p} n_T \right)^{-1}, \] \tag{3.7}
\[ \epsilon_T = \frac{C_R}{r_j} \frac{d\sigma_{pT \to \bar{p}}}{dE_{\bar{p}}} \left( \frac{d\sigma_{pp \to \bar{p}}}{dE_{\bar{p}}} \right)^{-1}. \] \tag{3.8}

Gaisser et al. [1] evaluated \( \epsilon_T \) for proton and helium targets using the latest cosmic-ray data to estimate \( r_j \) and the “wounded nucleon model” to estimate \( \sigma_{pT \to \bar{p}} \). They obtained
\[ \epsilon_p = 1.20, \] \tag{3.9}
\[ \epsilon_{He} = 4.28. \] \tag{3.10}

The \( r_j \) factor they used also agrees with the precise measurement of HEAO-3 [32]. To obtain the final \( \epsilon \) factor using the above values and eq. (3.7), the ISM compositions \( n_T \) are taken to be the same as that taken in Chapter 2, i.e., \( n_p = 0.9 \) and \( n_{He} = 0.1 \). To use the unique ISM composition is very important because, as Gaisser et al. pointed out [1], the resultant \( \bar{p} \) flux is not dependent on the ISM composition very much if one uses the same ISM composition when one obtain the escape length and calculates the \( \bar{p} \) production rate. With the above ISM composition, the final \( \epsilon \) factor that represents the heavy nuclei contribution is
\[ \epsilon = 1.16. \] \tag{3.11}

### 3.4 SLB Solution for Antiprotons

The total IS \( \bar{p} \) source spectrum can be obtained by multiplying the source spectrum from \( pp \) collisions (thick solid line of Fig. 3.6) by the \( \epsilon \) factor (eq. (3.11)). The escape length \( \lambda_{esc} \) was already obtained in Chapter 2. As explained there, we use the escape length \( \lambda_{esc}(R, \beta) \) shown by eqs. (2.24)-(2.28) because only these fit the positron data (Chapter 5). Equilibrium \( \bar{p} \) flux can then be obtained by the propagation calculation as follows.

SLB solutions for antiprotons were shown in refs [49, 3, 66, 2]. Here we use the most general form that includes the third interaction of \( \bar{p} \)’s. Our calculation technique is similar to that shown in ref. [66], though our numerical calculation does not need an iteration process. From eq. (2.18) and eq. (3.5), the SLB solution for antiprotons is written as
\[ I_{\bar{p}}(E_{\bar{p}}) = \left( \frac{dE}{dx} \right)^{-1} \bigg|_{E=E_{\bar{p}}} \times \int_{E_{\bar{p}}}^{\infty} dE'_{\bar{p}} \left[ 2 \epsilon \int_{E_{th}}^{\infty} I_{\bar{p}}(E_p) \frac{d\sigma_{pp \to \bar{p}}}{dE_{\bar{p}}} dE_{\bar{p}} + \frac{\xi_{p \to \bar{p}}}{m_p} \int_{E_{\bar{p}}}^{\infty} I_{\bar{p}}(E''_{\bar{p}}) \frac{d\sigma_{pp \to \bar{p}}}{dE_{\bar{p}}} dE''_{\bar{p}} \right] \times \exp \left[ \int_{E_{\bar{p}}}^{E_{\bar{p}}'} dE''_{\bar{p}} \left\{ \left( \frac{dE}{dx} \right)^{-1} \bigg|_{E=E''_{\bar{p}}} \left( \frac{1}{\lambda_{esc}(E''_{\bar{p}})} + \frac{1}{\lambda_{p}(E''_{\bar{p}})} \right) \right\} \right], \] \tag{3.12}
where $\lambda_{\bar{p}}$ is the total interaction length related to the total interaction cross section by eq. (2.16) and eq. (2.17), $d\sigma_{\bar{p}p}/dE_{\bar{p}}$ the differential cross sections of $\bar{p}$ production from the interactions of higher energy $\bar{p}$’s and ISM protons, and $\xi_{\bar{p}p}$ the ISM factor of this cross section given by eq. (2.14). Hereafter, $\bar{p}$’s directly produced from protons and nuclei will be referred to as “secondary $\bar{p}$’s,” while the $\bar{p}$’s produced from the $\bar{p}p$ interactions are called “tertiary $\bar{p}$’s.” The concerning $\bar{p}$ interactions will be determined and explained in the next two sections.

Eq. (3.12) is integrated numerically. In eq. (3.12), $I_{\bar{p}}$ does not seem to be calculable by an explicit integral because $I_{\bar{p}}$ is present on both the right- and left-hand sides. However, $I_{\bar{p}}(E_{\bar{p}})$ is dependent only on $I_{\bar{p}}(E_{\bar{p}}'')$, where $E_{\bar{p}}'' > E_{\bar{p}}$, because tertiary $\bar{p}$’s are only produced from higher energy secondary $\bar{p}$’s. Therefore the numerical integral of eq. (3.12) does not need the iteration process. The energy axis is first divided into small bins and the $I_{\bar{p}}$ of the highest energy bin is first obtained, while that of the second-highest energy bin is subsequently obtained using the value of $I_{\bar{p}}$ of the highest energy bin, and so on.

### 3.5 Antiproton Interaction Cross Sections

The total interaction length of $\bar{p}$’s in eq. (3.12) $\lambda_{\bar{p}}$ is related to the total interaction cross sections by eq. (2.16) and eq. (2.17), i.e.,

$$\frac{1}{\lambda_{\bar{p}}} = \frac{\xi_{\bar{p}}\sigma_{pp}}{m_p},$$  \hspace{1cm} (3.13)

$$\xi_{\bar{p}} = \left(\frac{\sum T \sigma_{\bar{p}T}}{\sigma_{\bar{p}p}}\right)\left(\frac{\sum T m_T}{m_p}\right)^{-1},$$  \hspace{1cm} (3.14)

where $\sigma_{\bar{p}p}$ and $\sigma_{\bar{p}T}$ are the total interaction cross sections of $\bar{p}$’s with ISM protons and $T$ (including protons and helium nuclei) targets respectively. In eq. (3.14), $\xi_{\bar{p}}$ is not dependent on energy, because the $\bar{p}T$ interaction cross sections are given by the geometrical approximation, i.e.,

$$\sigma_{\bar{p}T} = \sigma_{\bar{p}p}A_T^{2/3},$$  \hspace{1cm} (3.15)

where $A_T$ is the mass number of $T$ target. Note that this $A^{2/3}$ dependence of $pT \to$ total and inelastic cross sections was confirmed by the experimental data [67].

The $\bar{p}p$ interactions include elastic ($\bar{p}p \to \bar{p}p$) and inelastic interactions where the inelastic interactions include non-annihilation inelastic interaction ($\bar{p}p \to \bar{p}pX$) and annihilation ($\bar{p}p \to ann.$). The elastic cross section $\sigma_{\bar{p}p}p \to \bar{p}p$ accounts for 20 to 30 % of the total cross section [68]. However $\bar{p}$’s are expected to lose little energy by elastic scattering because the angular distribution of the final state $\bar{p}$’s in CMS has a sharp peak at zero angle (forward) [71]. On the other hand, in non-annihilation inelastic interactions, the CMS angular distributions of final state $\bar{p}$’s have broader peaks at zero angle with tails in large angle (backward) regions [71, 72, 75]. Since these tails correspond to the low energy region in LS, non-annihilation inelastic interactions are possible sources of low energy $\bar{p}$’s. Thus this interaction must be treated carefully.
in order to take into account the production of tertiary $\bar{p}$'s having lower energies as well as the loss of secondary $\bar{p}$'s. At energies lower than 1 GeV, the annihilation mode dominates.

Based on the above arguments, we consider only the annihilation and non-annihilation inelastic interactions as $\bar{p}$ loss processes, and only the non-annihilation inelastic interactions as the process of tertiary $\bar{p}$ production, i.e.,

$$\sigma_{\bar{p}p} = \sigma_{\bar{p}p\rightarrow inel.} = \sigma_{\bar{p}p\rightarrow ann.} + \sigma_{\bar{p}p\rightarrow \bar{pp}X},$$ (3.16)

$$\frac{d\sigma_{\bar{p}p\rightarrow \bar{p}}}{dE_{\bar{p}}} = \frac{d\sigma_{\bar{p}p\rightarrow \bar{pp}X}}{dE_{\bar{p}-f}},$$ (3.17)

where $\bar{p} - f$ means the final state $\bar{p}$.

As for $\sigma_{\bar{p}p\rightarrow inel.}$ and $\sigma_{\bar{p}p\rightarrow \bar{pp}X}$, we utilized the parameterization by Tan and Ng [66], which is based on and closely fits the experimental data [68]. However, the differential cross sections $d\sigma_{\bar{p}p\rightarrow \bar{pp}X}/dE_{\bar{p}-f}$ have thus far only been roughly treated. Namely, the simple approximation was often taken [66, 2, 3], i.e.,

$$\frac{d\sigma_{\bar{p}p\rightarrow \bar{pp}X}}{dE_{\bar{p}-f}} = \frac{\sigma_{\bar{p}p\rightarrow \bar{pp}X}}{E_{\bar{p}-i}},$$ (3.18)

where $E_{\bar{p}-i}$ is kinetic energy of the initial state $\bar{p}$. In this approximation, final state $\bar{p}$ has an energy between 0 and $E_{\bar{p}-i}$ with an equal probability per $dE_{\bar{p}-f}$ (dashed lines of Fig. 3.9, hereafter called “box-shape” approximation). The actual energy distribution of final state $\bar{p}$, however, should have a peak at $E_{\bar{p}-i}$ (solid lines of Fig. 3.9, explained later), because the “diffractive” feature of non-annihilation inelastic interaction was confirmed by a number of experiments [71, 72, 73, 74, 75, 76, 77, 78]. By using these data, we obtained an empirical form of the differential non-annihilation cross sections in the next section.

The ISM factor $\xi_{\bar{p}\rightarrow \bar{p}}$ is defined by eq. (2.14). Here it is reduced to

$$\xi_{\bar{p}\rightarrow \bar{p}} = \left(\sum_{T}^{ISM} \frac{\sigma_{\bar{p}T\rightarrow \bar{p}}}{\sigma_{\bar{p}p\rightarrow \bar{p}}} n_{T}\right) \left(\sum_{T}^{ISM} \frac{m_{T}}{m_{\bar{p}}} n_{T}\right)^{-1}. $$ (3.19)

The $\xi_{\bar{p}\rightarrow \bar{p}}$ factor is not dependent on energy, because we again use the geometrical approximation, i.e.,

$$\sigma_{\bar{p}T\rightarrow \bar{p}} = \sigma_{\bar{p}p\rightarrow \bar{p}} A_{T}^{2/3}. $$ (3.20)

The calculations of both ISM factors, i.e., $\xi_{\bar{p}}$ and $\xi_{\bar{p}\rightarrow \bar{p}}$, were performed using the same ISM composition, i.e., 90 % hydrogen and 10 % helium atoms.
3.6 Empirical Cross Sections of $\bar{p}p \to \bar{p}pX$ Interactions

In this section, the differential non-annihilation cross sections $d\sigma_{\bar{p}p \to \bar{p}pX}/dE_{\bar{p} - f}$ were calculated using the existing experimental data. Since the total cross sections $\sigma_{\bar{p}p \to \bar{p}pX}$ were already parameterized by Tan and Ng [66], which closely fits the data [68], we utilized this. We calculated only the energy ($E_{\bar{p} - f}$) spectra of the differential cross sections and normalized them to the total cross sections of Tan and Ng.

Among $\bar{p}p \to \bar{p}pX$ interactions, the dominant mode above $\sim 1$ GeV is pion production, i.e., $\bar{p}p \to \bar{p}pm\pi (m = 1, 2, \ldots)$ [76, 77]. The cross sections of kaon production and other modes are very small [77, 73]. At energies below $\sim 1$ GeV, charge exchange mode $\bar{p}p \to \bar{n}n \to \bar{p}pe^-e^+\nu\nu$ dominates [69, 70]. This mode, however, is not very important because $\bar{n}$'s are emitted forward in CMS [70] and most final state $\bar{p}$'s, which are decay products of $\bar{n}$'s, have energies close to the initial state $\bar{p}$'s. Our calculations include the above modes, i.e., the pion production modes and the charge exchange mode.

The existing data of $\bar{p}p \to \bar{p}pm\pi$ interactions [71, 72, 73, 74, 75, 76, 77, 78] are well explained by the one-pion-exchange model (OPEM) [79]. In OPEM, the energy and angular distribution of final state $\bar{p}$'s and pions are well explained by considering the following processes (Fig. 3.7): (i) one pion is exchanged between the initial state $\bar{p}$ and $p$, (ii) $\bar{p}$ and/or $p$ are excited into a heavy baryon state such as $\Delta(1231\text{ MeV})$, and (iii) the heavy baryons decay into nucleons and pions.

Although we did not employ OPEM directly, our empirical fitting was carried out based on the above processes. Among a number of heavy baryon resonance states, the dominant one is $\Delta(1231\text{ MeV})$ [80] (P$_{33}$ resonance, simply denoted as “$\Delta$” hereafter). The production of $\Delta$ isobars in $\bar{p}p \to \bar{p}pm\pi$ interactions was observed by a number of experiments [71, 72, 73, 74, 75, 76, 77, 78] over a wide energy range of $1\text{ GeV} < E_{\bar{p} - i} < 48\text{ GeV}$. Particularly, most of $\bar{p}p \to \bar{p}p\pi^+\pi^-$ interaction occurs via $\Delta$ state [72, 73, 74, 75, 76, 78], i.e., $\bar{p}p \to \Delta^{++}\Delta^{++} \to \bar{p}\pi^-p\pi^+$. Also in other modes, such as $\bar{p}p \to \bar{p}p\pi^0$, $\Delta$ production was observed [71, 73, 75] more clearly than any other resonance states. We took the approximation that $\bar{p}$ is always excited into $\Delta$ if kinematically allowed. The shape of resonance was reproduced from ref. [74].

The distribution of the transfer momentum squared $-t$ from $\bar{p}$ to $\Delta$ is dependent on incident energy $E_{\bar{p} - i}$. We reproduced the shape of $-t$ distribution at energies $E_{\bar{p} - i} = 0.925, 2.78, 4.84, 8.21, 31.1$, and $48.1\text{ GeV}$ from ref. [71], [73], [75], [76], [77], and [78] respectively.

The kinematics of $\Delta$ production are thus determined by resonance shape and $-t$ distribution as described above. The $\Delta$ isobar then decays into $\pi + \bar{p}$ [80]. The angular distribution of this two-body decay was assumed to be isotropic based on the experimental data $-\pi$ from $\Delta$ decay is almost isotropically emitted in the $\Delta$ rest system [72, 74, 75, 76].

In order to confirm whether our calculation correctly reproduces the energy distribution of final state $\bar{p}$'s, we calculated the CMS angular distribution of final state $\bar{p}$'s, $\bar{n}$'s, and $\bar{p}\pi$ systems and compared them with the data (Fig. 3.8). These CMS
angular distributions of antibaryons are closely related to the LS energy distribution of final state $\bar{p}$'s. Namely, the $\bar{p}$'s, $\bar{n}$'s, and $\bar{p}\pi$ systems emitted backward in CMS correspond to low energy $\bar{p}$'s in LS, while those emitted forward correspond to $\bar{p}$'s having energies close to the initial energy $E_{\bar{p}-i}$. If the CMS angular distribution agrees with the data, we can expect the LS energy distribution of $\bar{p}$'s to be correctly calculated with our present model. Of course, while it is better to directly compare the calculated LS energy distribution with the data, unfortunately, most experiments were carried out about 30 years ago, and the data of LS energy distribution of final state $\bar{p}$'s cannot be found anywhere today.

From here on, we will treat $\bar{p}$ and $\bar{n}$ together because their mass difference is negligible compared with incident energy or momentum transfer. Fig. 3.8 (a) shows the CMS angular distribution of $\bar{p}$'s ($\bar{n}$'s) in the interactions $\bar{p}p \rightarrow \bar{p}p\pi^0$, $\bar{p}p \rightarrow \bar{p}n\pi^+$, and $\bar{p}p \rightarrow \bar{n}p\pi^-$ at $E_{\bar{p}-i} = 0.93$ GeV. The histogram represents the data from ref. [71]. At this energy, the above three modes are dominant. Although there exists one more single pion production mode $\bar{p}p \rightarrow \bar{n}n\pi^0$ that is difficult to detect experimentally, including this mode will not significantly change the energy distribution of final state $\bar{p}$'s because the nature of the above interaction is expected to be quite similar to that of $\bar{p}p \rightarrow \bar{p}p\pi^0$ interaction due to isospin symmetry. The solid line in Fig. 3.8 (a) is the result of calculation under the present model. As shown, it is in good agreement with the data. Other lines, i.e., dashed, dotted, and dot-dashed lines are the CMS angular distributions in the box shape approximation. Since the box shape approximation does not correspond to the unique CMS angular distribution, these lines were obtained assuming the following.
Figure 3.8: CMS angular distribution of final state $\bar{p}$, $\bar{n}$, and $\bar{p}\pi$ system in $\bar{p}p$ non-annihilation interactions. Histograms and error bars are data from (a) [71], (b) [72], (c) [75], and (d) [70]. Solid lines show the present results of the empirical model. Other lines are CMS angular distributions corresponding to the box shape approximation (see text).
Figure 3.9: LS energy distribution of final state $\bar{p}$’s in $\bar{p}p$ non-annihilation interactions. Solid lines show the present results of the empirical model. Dashed lines represent the box shape approximation. For both models, total cross sections are normalized to the parameterization by Tan and Ng [66].
• Dashed line: distribution of the transverse momentum of final state $\bar{p}$’s $P_t$ is assumed to be $\propto \exp\{-P_t/(0.4 \text{ GeV}/c)\}$, which is generally shown as distribution in the hadron interactions [65].

• Dotted line: LS angular distribution is assumed to be $d\sigma/d\cos\theta = \text{const.}$ within a range kinematically allowed.

• Dot-dashed line: CMS energy of $\bar{p}$ is assumed to be unchanged in the interaction, i.e., an elastic approximation.

As shown in the figure, none of these agreed with the data. It was found that, in the box shape approximation, one always overestimates the yield of the backward $\bar{p}$’s.

Fig. 3.8 (b) and (c) are the same comparison at higher incident energies. Also at these energies, the present model well fits the data, but the box shape approximation overestimates the backward $\bar{p}$ yield. Fig. 3.8 (d) is the same comparison for the charge exchange interaction $\bar{p}p \rightarrow \bar{n}n$ at $E_{\bar{p}-i} = 0.23 \text{ GeV}$. At such low energies, the charge exchange interaction is the dominant mode. Again it is shown that our model well fits the data, but the box shape approximation overestimates low energy $\bar{p}$’s.

Finally, Fig. 3.9 shows the distribution of the LS kinetic energy of final state $\bar{p}$’s $E_{\bar{p}-f}$. The solid lines are the results of our model and dashed lines represent the box shape approximation. As shown in the figure, low energy $\bar{p}$’s are always overestimated in the box shape approximation. These differences in the differential cross sections of $\bar{p}$ non-annihilation interactions resulted in a considerable difference in the resultant $\bar{p}$ flux as shown in the next section.
3.7 Interstellar $\bar{p}$ Flux and the Previous Works

Fig. 3.10 shows the resultant IS $\bar{p}$ flux. The thick solid line is the present result. The thick dashed line is the result of calculation using the box shape approximation, while the thick dotted line represents the flux of secondary $\bar{p}$'s not including tertiary $\bar{p}$'s. As shown in the figure, tertiary $\bar{p}$'s are not negligible at low energies, but they are overestimated if one uses the box shape approximation. Therefore, the new parameterization of $\bar{p}$ non-annihilation cross section performed here is very important to estimate the most plausible spectral shape of low-energy $\bar{p}$ flux. However, the large discrepancy in the $\bar{p}$ flux at energies below 500 MeV is not very important because of the difficulty of observing them at 1 AU (at TOA) due to the solar modulation effect. As shown in Appendix B, the modulation parameter multiplied by the particle charge $Z\phi$ is approximately the energy loss in the Solar system, where $\phi$ is as large as $\sim 500$ MV even at the solar minimum as shown in Fig. 3.3, corresponding to the energy loss of $\sim 500$ MeV for $\bar{p}$'s ($Z = 1$).

The thick dot-dashed line is the $\bar{p}$ flux (secondary only) calculated using $\lambda_{esc}(R)$. This flux is significantly smaller than that obtained using $\lambda_{esc}(R, \beta)$. This discrepancy would remain as an uncertainty in the $\bar{p}$ flux if one used the nuclei data only to obtain $\lambda_{esc}$. However, since only $\lambda_{esc}(R, \beta)$ fits the positron data (Chapter 5), we can reject the possibility that $\bar{p}$ flux is so small as the thick dot-dashed line.

The thin lines are from previous works [1, 2, 3, 4]. The inputs used in their calculations are shown in Table 3.2. Among these previous results, the $\bar{p}$ flux of Webber et al. is the largest, while that of Simon et al. is the smallest. The causes of these discrepancies are shown in Table 3.3. As shown there, the large flux of Webber et al. is mainly due to (i) large proton flux (Fig. 3.4), (ii) the large contribution of heavy cosmic rays and targets (large $\epsilon$ factor), and (iii) box shape approximation for tertiary $\bar{p}$ production. The third only affects low energy $\bar{p}$'s, while the large proton flux and large $\epsilon$ factor increase the $\bar{p}$ flux over the whole energy region. On the other hand, the small flux of Simon et al. is mainly due to the small escape length and partially due to neglecting tertiary $\bar{p}$'s. Their flux is close to that shown by the thick dot-dashed line in Fig. 3.10, which was obtained by the calculation using $\lambda_{esc}(R)$ and neglecting tertiary $\bar{p}$'s.
Figure 3.10: Interstellar $\bar{p}$ flux calculated using the SLB model. Thick solid line is the present result. Other thick lines are for comparison (see text). Thin lines are the previous works by Stephens et al. [3] (dotted line), Webber et al. [2] (dashed line), Simon et al. [4] (dot-dashed line), and error bounds by Gaisser et al. [1] (solid lines).
Table 3.2: Inputs of $\bar{p}$ flux calculation used in the previous works (see also Table 3.1). As for $I_p$ and $\lambda_{esc}$, see also Fig. 3.4 and 2.4, respectively.

<table>
<thead>
<tr>
<th></th>
<th>$I_p$</th>
<th>$\sigma_{pp-\bar{p}}$</th>
<th>$\epsilon$</th>
<th>$\lambda_{esc}$</th>
<th>$\sigma_{pp}$</th>
<th>$d\sigma_{pp-\bar{p}}/dE_{\bar{p}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaisser et al. [1]</td>
<td>bounds</td>
<td>1.17</td>
<td>bounds</td>
<td>T [66]</td>
<td>neglected</td>
<td></td>
</tr>
<tr>
<td>Present work</td>
<td>LEAP [58]</td>
<td>T [65]</td>
<td>1.16</td>
<td>$\lambda_{esc}(R, \beta)$</td>
<td>T [66]</td>
<td>empirical</td>
</tr>
</tbody>
</table>

Table 3.3: Causes of discrepancies between present and previous results in $\bar{p}$ flux calculations.

<table>
<thead>
<tr>
<th></th>
<th>$I_p$</th>
<th>$\sigma_{pp-\bar{p}}$</th>
<th>$\epsilon$</th>
<th>$\lambda_{esc}$</th>
<th>Tertiary</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Webber et al. [2]</td>
<td>1 GeV</td>
<td>1.6</td>
<td>1.2</td>
<td>1.37</td>
<td>0.6</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>0.5 GeV</td>
<td>1.6</td>
<td>1.2</td>
<td>1.37</td>
<td>0.8</td>
<td>1.3</td>
</tr>
<tr>
<td>Simon et al. [4]</td>
<td>1 GeV</td>
<td>1.0</td>
<td>1.0</td>
<td>1.08</td>
<td>0.55</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>0.5 GeV</td>
<td>1.0</td>
<td>1.0</td>
<td>1.08</td>
<td>0.5</td>
<td>0.6</td>
</tr>
</tbody>
</table>

3.8 Solar Modulation and Comparison with the Data

In order to compare the present result with observations, the IS flux was modulated to the flux at 1 AU using the same model as proton modulation (see section 3.1). Fig. 3.11 shows the flux at 1 AU (at TOA) for various levels of the solar activity $\phi$. BESS 93 and 94 flights respectively correspond to $\phi = 700$ and 600 MV from the proton flux fitting (section 3.1). As shown in the figure, antiproton flux does not change very much with variations of the solar activity because of the spectral shape of antiprotons as pointed out by Labrador and Mewaldt [15]. According to Gleeson and Axford [117] (see also Appendix B), the flux at 1 AU $I_{1AU}$ and that at interstellar space $I_{IS}$ is approximately related as

$$\frac{I_{1AU}(E_{1AU})}{(E_{1AU} + m)^2 - m^2} = \frac{I_{IS}(E_{IS})}{(E_{IS} + m)^2 - m^2};$$

$$E_{1AU} = E_{IS} - Z\phi,$$

where $Z$ and $m$ respectively are the charge and mass of the particle, and $E_{1AU}$ and $E_{IS}$ respectively are the kinetic energy at 1 AU and interstellar space. Eq. (3.21)
corresponds to the Liouville theorem, and shows that the energy loss and the flux reduction are related. At the solar maximum (large $\phi$), cosmic rays lose a large amount of energy and the flux at 1 AU is strongly reduced compared with the IS flux. On the other hand, at the solar minimum (small $\phi$), flux reduction and energy loss are both small so that the observed flux is closer to the IS flux. Because IS $\bar{p}$ flux increases with increasing energy below 2 GeV, the two effects cancel each other resulting in unchanged flux at 1 AU. On the other hand, because IS $p$ flux decreases with energy, both the energy loss and flux reduction result in reducing the flux at 1 AU. Therefore, the proton flux at solar maximum at 1 AU is much smaller than that at solar minimum as shown in Fig. 3.11.

From the above effects, the $\bar{p}$ flux at 1 AU (at TOA) is shown to be more stable against solar modulation than $\bar{p}/p$ flux ratio at TOA. This is why we used $\bar{p}$ flux at TOA for the comparison between the present calculation and observations by the BESS experiment (Fig. 3.12). As shown, the BESS ’93 + ’94 data [13] are in good agreement with the prediction of the SLB model. This is a very important result which indicates that

- The main component of cosmic-ray antiprotons is probably the secondary product from the interactions between primary cosmic rays and the interstellar matter, although the minor contribution from primary $\bar{p}$’s cannot be excluded with the present precision of calculations as well as that of the data.

- The origin and propagation history of the dominant proton component is essentially the same as that of nuclei component.

The dotted lines in Fig. 3.12 show the error bounds of the present calculation associated with the errors in the escape length [33] and $\bar{p}$ production cross sections (Fig. 3.5). At present, those errors are smaller than the errors of the data. Therefore, further precision tests of the SLB model are possible with future observations of $\bar{p}$ flux using larger statistics.

Fig. 3.13 shows the $\bar{p}/p$ flux ratio at TOA. The data of the BESS experiment agree with the present calculation at this ratio as well. Although more data exist in the form of the $\bar{p}/p$ flux ratio, the other data cannot easily be interpreted because of the solar modulation effect. This ratio changes year by year due to solar modulation because the denominator $p$ flux varies widely as described above. For comparison between the data and calculations, therefore, it is necessary to carry out the compilation of the data of modulation levels corresponding to every flight. See ref. [15] for such analyses, where the interstellar $\bar{p}$ flux of Webber et al. [2] and that of Gaisser et al. [1] were tested.
Figure 3.11: $\bar{p}$ and $p$ flux at TOA for various levels of solar activity. Interstellar $\bar{p}$ and $p$ fluxes are respectively given by Fig. 3.10 (thick solid line) and eq. (3.1). Solar modulation calculations were performed using the spherically symmetric diffusion-convection model (Appendix B).
Figure 3.12: $\bar{p}$ flux at TOA: observations and prediction of the SLB model. Solid and dashed lines are the same as solid lines in Fig. 3.11 (Fisk’s solution, see also Appendix B). Dotted lines are the error bounds of SLB calculation associated with errors of the escape length [33] and those of the $\bar{p}$ production cross sections (Fig. 3.5). Data points are from the BESS experiment [13] (filled squares), Bogolomov et al. [20, 21] (open triangles), and Golden et al. [22] (filled triangles).
Chapter 3. Calculation of Cosmic-Ray Antiproton Flux using the SLB Model

Figure 3.13: $\bar{p}/p$ flux ratio at the top of the atmosphere. The lines are obtained from $\bar{p}$ and $p$ fluxes shown by the solid lines of Fig. 3.11. Data points are the same as Fig. 1.1, i.e., from the BESS experiment [13] (filled squares), IMAX experiment [14] (filled circles), PBAR experiment [16] (open diamonds), LEAP experiment [17, 18] (open circles), Buffington et al. [19] (open square), Bogolomov et al. [20, 21] (open triangles), and Golden et al. [22] (filled triangles).
Chapter 4

Antiproton Propagation in the Diffusive Reacceleration Model

4.1 The Reacceleration Models

In the current understanding, cosmic rays are considered to be “preaccelerated” at the source region. Although the source object and its acceleration mechanism has not yet been completely understood, it is known that the shock wave acceleration at the supernova remnants (SNR’s) (see [10] for a review) can reproduce the source spectrum as well as the source intensity that is consistent with observations, when considered under the standard propagation theory (e.g., SLB model). On the other hand, it is not well known whether or not the “reacceleration” during propagation occurs in interstellar space. Since propagation models with no reacceleration, such as the SLB model, can explain the exiting comic-ray data as shown in the previous chapters, reacceleration does not seem to be necessary to consider. However, as pointed out in refs. [81], [23], [24], and [42], the second-order Fermi type of acceleration (simply called “Fermi acceleration” hereafter) [82] should inevitably occur during diffusive propagation in the Galaxy. Fermi acceleration is thus considered to be the minimal reacceleration process. In other words, Fermi acceleration probably occurs during the diffusive propagation, and it is a matter of degree. Per Heinbach and Simon [24], we call the propagation model with Fermi acceleration the “Diffusive Reacceleration (DR) model.” We will test this model by antiproton and positron fluxes in this and next chapters respectively.

As shown by Simon et al. [81], the DR model may solve some problem of the SLB model. In the SLB model, the energy-dependent escape length falls at low energies as shown in Chapter 2. Although we have shown that this fall is a function of particle velocity not being a function of rigidity (Chapter 5), the origin of this fall is unknown, and the low energy escape length was obtained with purely empirical fitting. On the other hand, in the DR model [23, 24], the cosmic-ray data can be reproduced with the escape length expressed as a single power of rigidity down to the lowest energy of which the data are available. In the DR model, low-energy fall of the B/C flux ratio is not reproduced by the fall of escape length but naturally reproduced by the reacceleration effect. Briefly, low-energy borons are depleted because they are shifted to higher energies, while low-energy carbons are less shifted.
because they primaries are younger than secondary borons [81].

Further, the power exponent of the escape length under the DR model can be taken to be $-1/3$, which would indicates a Kolmogorov-type spectrum of magnetic turbulence. Compared with the exponent in the SLB model $-0.65$ (eq. (2.26) and (2.30)), the exponent in the DR model $-1/3$ corresponds to weaker energy dependence of the escape length. This also originates from the reacceleration effect. For this effect, a comprehensible explanations were shown in refs. [81] and [42]. Briefly, in the DR model, the energy dependence of the escape length is not necessary to be so steep as the observed secondary to primary ratios (e.g., boron to carbon ratio), because energy dependences of secondary to primary ratios are partially reproduced by the effect that equilibrium spectra of cosmic ray nuclei (e.g., borons) naturally become softer (steeper) than the source spectra (e.g., carbons are the sources of borons) since the reacceleration is more effective at lower energies. Such “weak acceleration” nature arises from the feature of second-order Fermi acceleration, i.e., (i) energy gain is proportional to the total energy (i.e., proportional to the rest mass if particles are non-relativistic) and (ii) degree of reacceleration is dependent on the rate of scattering, i.e., the inverse of the diffusion coefficient. Details will be shown in the following sections.

In the previous works of the DR model [24, 42], the Leaky-Box approximation was taken, i.e., the injection of cosmic rays, their escape, interaction with ISM, ionization energy loss, and the reacceleration were treated as occurring simultaneously. We have shown in Chapter 2 that this approximation is valid for the stable nuclei when considered under the simple Diffusion model. However, the validity of this approximation under the DR model have not yet confirmed. As pointed out by Seo and Ptuskin [42], the validity of the Leaky-Box approximation for the DR model is dependent on the size of the region where the reacceleration occurs. If the reacceleration region is much thinner than the total confinement volume, the Leaky-Box approximation was proved to be valid [42]. For example, supposing that the half thickness of the disk is 100 pc, that of reacceleration region is 300 pc, and that of total confinement volume (galactic halo) is 2 kpc. In this case, all processes including reacceleration occur within 300 pc, and cosmic rays will traverse this region many times before escape. In addition, reacceleration and interactions are so slow that the rate of them can be averaged over the life time of cosmic rays. As a result, one can treat all processes, i.e., reacceleration, interactions, and escape, as if they occurred simultaneously.

However, we naively expect that reacceleration occurs throughout the confinement volume, because it is a part of the diffusion process itself as emphasized by Heinbach and Simon [24]. In such situation, the Leaky-Box approximation is not easily proved to be valid because the reacceleration does not occur simultaneously with the interactions with ISM. The reacceleration occurs everywhere, while injection of cosmic rays, interaction with ISM, and ionization energy loss occur only in the galactic disk. Therefore, the propagation history of cosmic rays is, in principle, dependent on the relative thickness of the halo to disk. To examine such dependence, here we use the 1-D halo model being similar to the 1-D Diffusion model, and argue the relation between the halo thickness and reacceleration effects.
In addition, we reexamine the scattering process by the moving magnetic field. In ref. [82], Fermi argued the scattering of cosmic-ray particle by the magnetic field line which is moving forward or backward (called “1-D scattering model”). This model was used by Heinbach and Simon [23, 24]. Here we use a 3-D expansion of this model (called “3-D scattering model”) where the field line is moving with random direction in a 3-D momentum space. As shown later, the low-energy antiproton flux in the 3-D scattering model is considerably smaller than that in the 1-D scattering model.

4.2 The Present Model

As described above, the novel features of the present model are (i) 3-D scattering model, and (ii) 1-D halo model, which are explained in order.

Interactions of cosmic rays and turbulent magnetic fields have been considered by a number of authors. Particularly, concerning the diffusion approximation and its application to the galactic propagation of cosmic rays, see ref. [10]. Although Seo and Ptuskin [42] studied the reacceleration model based on such arguments using magnetohydrodynamics (MHD), here we take more simple argument using the 3-D scattering model as follows.

Probability per unit time that a cosmic-ray particle collides with a turbulent magnetic fields (simply called “turbulence” hereafter) moving with a given direction is written as

$$d\Gamma(\theta_{rel}) = \Gamma_0 \frac{\gamma_{rel} \beta_{rel} \cos \theta_{rel}}{\gamma_B \beta} \cdot 2,$$

(4.1)

where $\beta$ is the speed of cosmic-ray particle in laboratory system (LS) divided by the speed of light and $\gamma = (1 - \beta^2)^{-1/2}$ the corresponding Lorentz factor, $\beta_B$ and $\gamma_B$ those for the turbulence, $\beta_{rel}$ and $\gamma_{rel}$ those for the relative motion of the cosmic-ray particle and turbulence, $\theta_{rel}$ the angle between the directions of motions of the cosmic-ray particle and turbulence in LS (Fig. 4.1), and $\Gamma_0$ the collision probability if the turbulence were at rest. The following relations are easily obtained by Lorentz transformations;

$$\beta_{rel} = \frac{1}{1 - \beta_B \beta_B} \left( \frac{\beta_B}{\gamma_B} \right)^2 + (\beta_B - \beta_{rel})^2,$$

(4.2)

$$\gamma_{rel} = (1 - \beta_B \beta_B) \gamma_B,$$

(4.3)

where $\beta_{\parallel} \equiv \beta \cos \theta_{rel}$ and $\beta_{\perp} \equiv \beta \sin \theta_{rel}$. Note that the collision probability of eq. (4.1) is mainly determined by the relative velocity $\beta_{rel}$.

The above estimation of the collision probability neglected the spiral motion of cosmic-ray particles. This is valid if the size of the turbulence is similar to the radius of spiral motion (Larmor radius) because, in such situation, the spiral motion will be disturbed by the turbulence before collision occurs so that the motion of cosmic-ray particle can be approximated by random walk. For the condition that collision occurs, the wavelength of the turbulence should be of the same order as Larmor radius. Therefore, the condition that the spiral motion is not effective is that the size and wavelength of the turbulence is of the same order. This situation is
Figure 4.1: Scattering of cosmic-ray particle with moving turbulence.
very likely to be the case because the turbulences are expected to be in equilibrium by the energy transport from larger turbulences to smaller ones (e.g., Kolmogorov turbulence). Therefore, we did not consider the spiral motion of cosmic-ray particles.

In the turbulence rest system (denoted as “BS”), the cosmic-ray particle goes in with total energy (kinetic energy plus rest mass) and momentum,

\[ E_{\text{tot},i}^{BS} = m \gamma_{\text{rel}}, \tag{4.4} \]

\[ P_{i}^{BS} = m \gamma_{\text{rel}} \beta_{\text{rel}}, \tag{4.5} \]

where \( m \) is the mass of the particle. Then the particle is assumed to be scattered isotropically, i.e., the final state momentum parallel to the direction of boost of LS is

\[ P_{||,f}^{BS} = m \gamma_{\text{rel}} \beta_{\text{rel}} \cos \theta_{\text{sc}}, \tag{4.6} \]

where \( \theta_{\text{sc}} \) takes the random value between \(-1\) and \(1\) with an equal probability per \( d \cos \theta_{\text{sc}} \). The absolute values of total energy and momentum are unchanged from eqs. (4.4), and (4.5) because of the large recoil mass of the turbulence. Then the final state total energy of cosmic-ray particle in LS is

\[ E_{\text{tot},f}^{LS} = \gamma_{B}(E_{\text{tot},i}^{BS} - \beta_{B} P_{||,f}^{BS}) \]

\[ = E_{\text{tot},i}^{LS} \gamma_{B}^{2}(1 - \beta_{B} \beta_{\text{rel}} \cos \theta_{\text{rel}})(1 - \beta_{B} \beta_{\text{rel}} \cos \theta_{\text{sc}}), \tag{4.7} \]

where \( E_{\text{tot},i}^{LS} = m \gamma \) is initial state total energy of the cosmic-ray particle in LS. Note that the case \( \cos \theta_{\text{rel}} = -1 \) and \( \cos \theta_{\text{sc}} = -1 \) corresponds to the “head-on collision,” while the case \( \cos \theta_{\text{rel}} = 1 \) and \( \cos \theta_{\text{sc}} = 1 \) corresponds to the “overtaking collision,” where the final state energies are respectively,

\[ E_{\text{tot}}^{\text{max}} = E_{\text{tot},i}^{LS}(1 + 2 \beta_{B} \beta_{\text{rel}}^{2} + \beta_{B}^{2})/(1 - \beta_{B}^{2}), \tag{4.8} \]

\[ E_{\text{tot}}^{\text{min}} = E_{\text{tot},i}^{LS}(1 - 2 \beta_{B} \beta_{\text{rel}}^{2} + \beta_{B}^{2})/(1 - \beta_{B}^{2}), \tag{4.9} \]

which agree with the results shown by Fermi [82].

The total collision probability per unit time is obtained from eq. (4.1), i.e.,

\[ \Gamma_{\text{col}} = \int_{-1}^{1} d\Gamma(\theta_{\text{rel}}) d \cos \theta_{\text{rel}} \]

\[ = \Gamma_{0} \left\{ \frac{1 + \beta^{2}}{2 \beta^{2}} + \frac{(1 - \beta^{2})(1 - \beta_{B}^{2})}{4 \beta^{2} \beta_{B}} \ln \frac{1 - \beta_{B}}{1 + \beta_{B}} \right\} \tag{4.10} \]

\[ \simeq \Gamma_{0} \left( 1 + \frac{1}{3} \frac{\beta_{B}^{2}}{\beta^{2}} - \frac{1}{3} \beta_{B}^{2} \right). \tag{4.11} \]

The last line (4.11) is obtained using the approximation that \( \beta_{B} \ll \beta \).

When the particle with initial state total energy \( E_{\text{tot}} \) is scattered, the probability that final state total energy falls between \( E'_{\text{tot}} \) and \( E'_{\text{tot}} + dE'_{\text{tot}} \) is obtained using
DR model with 1-D halo is written as

\[ \Theta(E_{tot}', E_{tot}) = \frac{1}{\Gamma_{\text{col}}} \int_{-1}^{1} d \cos \theta_{\text{rel}} \frac{d \Gamma(\theta_{\text{rel}})}{d \cos \theta_{\text{rel}}} \int_{-1}^{1} d \cos \theta_{\text{sc}} \frac{\delta(E_{tot}' - E_{tot,f}^{LS})}{2} \]

They are obtained using eqs. (4.1), (4.7) and (4.11) as

\[ \frac{1}{\Gamma_{\text{col}} 4E_{tot} \beta B} \left[ 1 + \frac{1}{\beta B} \left( \frac{E_{tot}'}{E_{tot}} - 1 \right) + \sqrt{\frac{1}{\beta^2} \left( \frac{E_{tot}'}{E_{tot}} \right)^2 - 1} \right] \]

\[ \left( E_{tot}^{\text{min}} < E_{tot}' < E_{tot} \right) \quad (4.12) \]

\[ \frac{1}{\Gamma_{\text{col}} 4E_{tot} \beta B} \left[ 1 - \frac{1}{\beta B} \left( \frac{E_{tot}'}{E_{tot}} - 1 \right) + \sqrt{\frac{1}{\beta^2} \left( \frac{E_{tot}'}{E_{tot}} \right)^2 - 1} \right] \]

\[ \left( E_{tot} < E_{tot}' < E_{tot}^{\text{max}} \right) \quad (4.13) \]

Using the above energy distribution, the cosmic-ray transport equation for the DR model with 1-D halo is written as

\[ \frac{\partial N_i}{\partial t} = D \frac{\partial^2 N_i}{\partial z^2} + \left( \frac{1}{\tau_i} + \frac{1}{\gamma \tau_{\text{dec}}} \right) N_i + \frac{\partial}{\partial E_i} \left( \frac{dE}{dt} N_i \right) \]

\[ + \Gamma_{\text{col}} N_i - \int_{0}^{\infty} \Gamma_{\text{col}}(E_i') \Theta(m + E_i', m + E) N_i(E_i') dE_i' = Q_i + S_i. \quad (4.14) \]

Compared with eq. (2.3), the above equation has two additional terms, i.e., the last two terms of left-hand side, which represents the reacceleration effects. The above equation is general form for the reacceleration model. However it is not necessary to treat this equation directly. We can take the Fokker-Plank approximation as normally done because the energy change per one collision is very small, i.e., at most \( E_{tot}/\beta B \) as shown in eqs. (4.8), (4.9), (4.12), and (4.13), where the velocity of the turbulence \( \beta B \) is at most \( \sim 10^{-4} \). By using Fokker-Plank approximation, we obtain

\[ \frac{\partial N_i}{\partial t} = D \frac{\partial^2 N_i}{\partial z^2} + \left( \frac{1}{\tau_i} + \frac{1}{\gamma \tau_{\text{dec}}} \right) N_i + \frac{\partial}{\partial E_i} \left( \frac{dE}{dt} N_i \right) \]

\[ + \frac{\partial}{\partial E_i} \left( \Gamma_{\text{col}} \langle \Delta E \rangle_{\text{col}} N_i \right) - \frac{\partial^2}{\partial E_i^2} \left( \frac{1}{2} \Gamma_{\text{col}} \langle (\Delta E)^2 \rangle_{\text{col}} N_i \right) = Q_i + S_i, \quad (4.15) \]

where \( \langle \Delta E \rangle_{\text{col}} \) and \( \langle (\Delta E)^2 \rangle_{\text{col}} \) respectively are the energy gain and dispersion per collision. They are obtained using eq. (4.1), (4.7) and (4.11) as

\[ \langle \Delta E \rangle_{\text{col}} = \frac{1}{\Gamma_{\text{col}}} \int_{-1}^{1} d \cos \theta_{\text{rel}} \frac{d \Gamma(\theta_{\text{rel}})}{d \cos \theta_{\text{rel}}} \int_{-1}^{1} d \cos \theta_{\text{sc}} \left( E_{tot,f}^{LS} - E_{tot,i}^{LS} \right) \]

\[ = \frac{E_{tot,i}^{LS}}{\Gamma_{\text{col}} 1 - \beta B^2} \left\{ \frac{\Gamma_0}{\Gamma_{\text{col}}} \frac{1}{1 - \beta B^2} \left( 1 + \frac{1}{3} \frac{\beta B^2}{\beta^2} \right) - 1 \right\} \]

\[ \simeq \frac{4}{3} E_{tot} \beta B^2, \quad (4.16) \]

\[ \langle (\Delta E)^2 \rangle_{\text{col}} = \frac{1}{\Gamma_{\text{col}}} \int_{-1}^{1} d \cos \theta_{\text{rel}} \frac{d \Gamma(\theta_{\text{rel}})}{d \cos \theta_{\text{rel}}} \int_{-1}^{1} d \cos \theta_{\text{sc}} \left( E_{tot,f}^{LS} - E_{tot,i}^{LS} \right)^2 \]

\[ = \frac{2}{3} E_{tot}^2 \beta^2 \beta B^2, \quad (4.17) \]
where \( E_{\text{tot}} \equiv E_{\text{tot},i}^{LS} \), and the nearly equal symbol means that the approximation that \( \beta_B \ll \beta \) was used.

The dispersion per collision \( \langle (\Delta E)^2 \rangle_{\text{col}} \) is related to the diffusion coefficient in energy space. Such energy space diffusion was most impressively shown by Heinbach and Simon [24], namely, an initial monochromatic test spectrum becomes broader with time (see Fig. 14 of ref [24]). As for \( \langle \Delta E \rangle_{\text{col}}, \langle (\Delta E)^2 \rangle_{\text{col}} > 0 \) means that the particle should gain energy statistically by repeating collisions. The causes of this energy gain are that (i) the large angle (\( \cos \theta_{\text{rel}} < 0 \)) collisions, in which the mean energy change is positive when averaged over \( \cos \theta_{\text{sc}} = -1 \) to 1 (eq. (4.7)), more frequently occur than small angle ones as shown in eq. (4.1), and (ii) the maximum energy gain \( (E_{\text{tot}}^{\text{max}} - E_{\text{tot},i}^{LS}, \text{eq. (4.8)}) \) is larger than the maximum energy loss \( (E_{\text{tot},i}^{LS} - E_{\text{tot}}^{\text{min}}, \text{eq. (4.9)}) \) by an order of \( \beta_B^2 \). Although this nature is essentially the same as that under the 1-D scattering model [82] used by Heinbach and Simon [24], the ratio of \( \langle \Delta E \rangle_{\text{col}} \) to \( \langle (\Delta E)^2 \rangle_{\text{col}} \) significantly differs as shown in Table 4.1. The last column of this table shows \( \langle (\Delta E)^2 \rangle_{\text{col}} \) to \( \langle \Delta E \rangle_{\text{col}} \) ratio represented as a dimensionless factor by multiplying \( E_{\text{tot}}^{-1} \). As shown, \( \langle \Delta E \rangle_{\text{col}} \) to \( \langle (\Delta E)^2 \rangle_{\text{col}} \) ratio in the 1-D scattering model is 2 times larger than that in the 3-D scattering model. This difference is important as shown in the section 4.5. Briefly, the secondary to primary nuclei ratios (such as B/C ratio) are mainly dependent on the energy gain \( \langle \Delta E \rangle_{\text{col}} \) because of their falling spectrum, while the antiproton flux is mainly dependent on dispersion \( \langle (\Delta E)^2 \rangle_{\text{col}} \) because antiproton spectrum has a peak at \( \sim 2 \) GeV. Therefore, \( \langle (\Delta E)^2 \rangle_{\text{col}} \) to \( \langle \Delta E \rangle_{\text{col}} \) ratio is very important for antiproton flux calculation because the absolute intensity of reacceleration is first tuned to fit the nuclei fluxes and then the antiproton flux is calculated.

Table 4.1: Energy gain and dispersion per collision with moving turbulence.

<table>
<thead>
<tr>
<th>Model</th>
<th>( \langle \Delta E \rangle_{\text{col}} )</th>
<th>( \langle (\Delta E)^2 \rangle_{\text{col}} )</th>
<th>( \langle (\Delta E)^2 \rangle_{\text{col}} / \langle \Delta E \rangle_{\text{col}} E_{\text{tot}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-D scattering model [24]</td>
<td>( 4E_{\text{tot}} \beta_B^2 )</td>
<td>( 4E_{\text{tot}} \beta_B^2 \beta^2 \beta_B^2 )</td>
<td>( \beta^2 )</td>
</tr>
<tr>
<td>3-D scattering model</td>
<td>( \frac{4}{3}E_{\text{tot}} \beta_B^2 )</td>
<td>( \frac{4}{3}E_{\text{tot}} \beta^2 \beta_B^2 \beta_B^2 )</td>
<td>( \frac{1}{2} \beta^2 )</td>
</tr>
</tbody>
</table>

### 4.3 Relation of Reacceleration and Diffusion

The diffusion coefficient and the intensity of Fermi acceleration is closely related as follows. The collision probability \( \Gamma_{\text{col}} \) in eq. (4.15) is given by

\[
\Gamma_{\text{col}} \simeq \Gamma_0 = \frac{\beta c}{l},
\]  

(4.18)

where \( l \) is the scattering mean free path. The mean free path and diffusion coefficient \( D \) are also related under the assumption of the isotropic scattering as

\[
D = \frac{1}{6} \beta cl.
\]  

(4.19)
The energy gain and dispersion per unit time are subsequently obtained using eq. (4.18), (4.19), (4.16), and (4.17) as

\[
\langle \frac{dE}{dt} \rangle_{\text{reac}} \equiv \langle \Delta E \rangle_{\text{col}} \Gamma_{\text{col}}
\]

\[
\simeq \frac{2}{9} \beta_B^2 E_{\text{tot}} \beta^2 c^2 \frac{1}{D}, \quad (4.20)
\]

\[
\langle (\Delta E)^2 / \Delta t \rangle_{\text{reac}} \equiv \langle (\Delta E)^2 \rangle_{\text{col}} \Gamma_{\text{col}}
\]

\[
\simeq \frac{1}{9} \beta_B^2 E_{\text{tot}}^2 \beta^4 c^2 \frac{1}{D}. \quad (4.21)
\]

With the confinement time in the Galaxy [10, 43]

\[
\tau_{\text{esc}} \approx h_h^2 \frac{2D}{\lambda_{\text{esc}}},
\]

we can finally evaluate the total amount of the energy gain and dispersion during the confinement in the Galaxy as

\[
\langle \Delta E \rangle_{\text{esc}} = \langle \frac{dE}{dt} \rangle_{\text{reac}} \tau_{\text{esc}} \sim \frac{1}{9} \beta_B^2 E_{\text{tot}} \beta^2 c^2 \left( \frac{h_h}{D} \right)^2, \quad (4.23)
\]

\[
\langle (\Delta E)^2 \rangle_{\text{esc}} = \langle \frac{(\Delta E)^2 / \Delta t}{\Delta t} \rangle_{\text{reac}} \tau_{\text{esc}} \sim \frac{1}{18} \beta_B^2 E_{\text{tot}}^2 \beta^4 c^2 \left( \frac{h_h}{D} \right)^2, \quad (4.24)
\]

where \( h_h \) is the half thickness of the galactic halo (Appendix C). From the above equations, we find that \( \langle \Delta E \rangle_{\text{esc}} \) and \( \langle (\Delta E)^2 \rangle_{\text{esc}} \) at a given energy are dependent only on \( \beta_B \) and the ratio \( h_h/D \), but are not dependent on \( h_h \) or \( D \) alone. Such dependence is the same as that of the escape length (Appendix C), i.e.,

\[
\lambda_{\text{esc}} = \rho_0 \beta c h_g \frac{h_h}{D}. \quad (4.25)
\]

In other words, \( \langle \Delta E \rangle_{\text{esc}} \) and \( \langle (\Delta E)^2 \rangle_{\text{esc}} \) are dependent only on \( \beta_B \) and \( \lambda_{\text{esc}} \). When considering such dependence, it seems to be appropriate to introduce the Leaky-Box approximation because both the reacceleration and the interactions with ISM seem to be governed by the traversed matter. However, although the final amounts \( \langle \Delta E \rangle_{\text{esc}} \) and \( \langle (\Delta E)^2 \rangle_{\text{esc}} \) are dependent only on the total traversed matter \( \lambda_{\text{esc}} \), the energy gain and dispersion at a certain age are dependent not only on the traversed matter until then but also on their age for the reason as follows. As discussed in the previous section, the reacceleration and interactions with ISM do not occur simultaneously, i.e., the reacceleration occurs everywhere in the Galaxy while the interactions with ISM occur at the galactic disk only. Since cosmic rays are injected in the disk, they experience the interactions with ISM at their young period, when they travel near the galactic plane, while they experience the reacceleration throughout their life. In such situation, the Leaky-Box approximation is not appropriate because it treats all the processes as if occurring simultaneously. Therefore, we use the 1-D halo model (eq. (4.15)) for all the calculations of fluxes of nuclei (section 4.4), antiprotons (section 4.5), and positrons (Chapter 5).
Chapter 4. Antiproton Propagation in the Diffusive Reacceleration Model

4.4 Fitting of Cosmic Ray Nuclei Data

The calculations of nuclei fluxes are performed in the same way as Chapter 2 but using the solution of the DR model shown in Appendix C. Parameters of the DR model are the escape length $\lambda_{esc}$, velocity of the turbulence $\beta_B$, and half thickness of the galactic halo $h_h$. The escape length represents the mean amount of the matter that cosmic rays would traverse if it were not for the reacceleration. However, since the energy of cosmic rays are always changing during the propagation and $\lambda_{esc}$ is energy dependent, one can not easily estimate actual amount of the traversed matter. The $\lambda_{esc}$ parameter is then simply considered to be the ratio of $h_h/D$ (eq. (4.25)). This ratio is related both to the interaction with ISM and reacceleration, then this ratio, namely $\lambda_{esc}$, is the most important parameter also in the DR model.

The velocity of the turbulence $\beta_B$ is the only parameter to determine the intensity of the reacceleration. Heinbach and Simon [24] represented the intensity of the reacceleration by the “$\eta$” parameter, while Seo and Ptuskin [42] represented it by the “$\alpha$” parameter. Unlike these parameters, our $\beta_B$ directly represents the speed of the turbulence.

The halo thickness $h_h$ is proportional to $D^{-1}$ if $\lambda_{esc}$ is fixed (eq. (4.25)), then $h_h$ determine the absolute value of the diffusion coefficient. Since the diffusion coefficient is related to the reacceleration rate (eq. (4.20) and (4.21)), the rate is consequently determined by $h_h$. The surviving fraction of the radioactive nuclei ($^{10}\text{Be}$) is also dependent on $h_h$ as explained in Chapter 2. We must examine whether such dependence of the $^{10}\text{Be}$ surviving fraction on $h_h$ is affected by the reacceleration effect or not.

Tuning of these parameters are performed as follows. First, we assume that $\lambda_{esc}$ is proportional to $R^{-1/3}$, which is equivalent to $l \propto R^{1/3}$ by eq. (4.19) and (4.25), and is expected when the spectrum of magnetic turbulence is Kolmogorov type. Next, the halo thickness $h_h$ is assumed. Finally, $\beta_B$, absolute value of $\lambda_{esc}$, and source spectrum as well as the relative source abundances of primary nuclei are tuned to fit the observed relative and absolute fluxes. This procedure is repeated with another $h_h$ value. The range of $h_h$ is examined where the $^{10}\text{Be}/^{9}\text{Be}$ flux ratio agrees with the data.

By the analyses described above, we have found that the data of $^{10}\text{Be}/^{9}\text{Be}$ flux ratio are consistent with $h_h$ between 1 to 4 kpc, the same range as that under the Diffusion model (Chapter 2). Calculated nuclei fluxes are shown in Fig. 4.2-4.7 for $h_h = 1, 2,$ and $4$ kpc.
Figure 4.2: Cosmic-ray nuclei flux: calculations under the DR model and observations. Data points and solar modulation calculations are the same as Fig. 2.1 and 2.2. Halo thickness, speed of the turbulent magnetic field, and the escape length respectively are $h_h = 1 \text{kpc}, \beta_B c = 53.1 \text{ km/s}$, and $\lambda_{esc} = 11.2 \times (R/GV)^{-1/3} \text{ gcm}^{-2}$. 
Figure 4.3: (Continued)
Figure 4.4: Cosmic-ray nuclei flux: calculations under the DR model and observations. Data points and solar modulation calculations are the same as Fig. 2.1 and 2.2. Halo thickness, speed of the turbulent magnetic field, and the escape length respectively are $h_h = 2$ kpc, $\beta_B c = 62.3$ km/s, and $\lambda_{esc} = 10.3 \times (R/GV)^{-1/3}$ gcm$^{-2}$.
Figure 4.5: (Continued)
Figure 4.6: Cosmic-ray nuclei flux: calculations under the DR model and observations. Data points and solar modulation calculations are the same as Fig. 2.1 and 2.2. Halo thickness, speed of the turbulent magnetic field, and the escape length respectively are $h_h = 4 \text{kpc}$, $\beta_B c = 62.3 \text{ km/s}$, and $\lambda_{esc} = 10.3 \times (R/GV)^{-1/3} \text{ gcm}^{-2}$. 
Chapter 4. Antiproton Propagation in the Diffusive Reacceleration Model

Figure 4.7: (Continued)
Figure 4.8: $^{10}$Be/$^9$Be ratio in the Diffusion and DR models. Data points and solar modulation calculations are the same as Fig. 2.1 and 2.2. In each model, the halo thickness is $h_h = 1, 2, 4$ kpc from above.

The other parameters of these three models were compiled in Table 4.2. As shown, the speed of turbulence $\beta_{BC}$ and the escape length $\lambda_{esc}$ are not dependent on the halo thickness $h_h$ very much. This is the results expected by the arguments in section 4.3. Only the $^{10}$Be/$^9$Be flux ratio is dependent on halo thickness. As shown in Fig. 4.8, the range of the halo thickness that is consistent with the data of $^{10}$Be/$^9$Be flux ratio is almost equal to that in the Diffusion model, although the energy dependence of this ratio is somewhat different.

Table 4.2: The obtained parameters of the DR model, i.e., the half thickness of the halo $h_h$, the speed of the turbulent magnetic field $\beta_{BC}$, and the escape length $\lambda_{esc}$.

<table>
<thead>
<tr>
<th>$h_h$ (kpc)</th>
<th>$\beta_{BC}$ (km/s)</th>
<th>$\lambda_{esc}$ (gcm$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>53.1</td>
<td>$11.2 \times (R/GV)^{-1/3}$</td>
</tr>
<tr>
<td>2</td>
<td>62.3</td>
<td>$10.3 \times (R/GV)^{-1/3}$</td>
</tr>
<tr>
<td>4</td>
<td>62.3</td>
<td>$10.3 \times (R/GV)^{-1/3}$</td>
</tr>
</tbody>
</table>

Fig. 4.9 (a) shows the obtained escape length under the DR model along with the previous works and that under the SLB model. Like the previous works [24, 42], our escape length under the DR model is in a single power low of rigidity with an exponent of $-1/3$, i.e, $\lambda_{esc} = \lambda_0 R^{-1/3}$, where $\lambda_0$ is a constant. Since we initially
Figure 4.9: Escape length and source spectra under the SLB and DR models. (a) Escape length under the SLB and DR models. Thick solid line; present result of the SLB model (see Chapter 2), thin solid line; result of the SLB model by Heinbach et al., thick dashed line; present results of the DR model with $h = 2$ and 4 kpc, which agree with the result of Heinbach et al. [24], thick dotted line; present result of the DR model with $h = 1$ kpc, and thin dashed line; result of the DR model by Seo et al. [42] $(14 \times (R/GV)^{-1/3})$. (b) Source spectra under the SLB and DR models.
assumed this functional form, we cannot conclude that this is the unique form to reproduce the observed nuclei fluxes under the DR model. It is necessary to test other possibilities, e.g., $\lambda_{\text{esc}} = \lambda_0 R^{-0.4}$, independently of previous authors. From the present results, it was confirmed that under the assumption that $\lambda_{\text{esc}} = \lambda_0 R^{-1/3}$ the nuclei fluxes can be reproduced with $\lambda_0 = 10 \sim 11 \text{ (gcm}^{-2})$, which agrees with the previous results [24, 42].

Fig. 4.9 (b) shows source spectra under the DR model along with that under the SLB model. As previously pointed out [24, 42], the source spectrum under the DR model is steeper at high energies due to flatter energy dependence of $\lambda_{\text{esc}}$.

4.5 Calculation of Antiproton Flux and Discussion

Using thus obtained models, antiproton fluxes are calculated as follows. The source spectrum of the antiprotons is the same as what is used in the SLB model (Chapter 3), i.e., the solid line of Fig. 3.6 multiplied by the factor $\epsilon = 1.16$ (eq. (3.11)). Using the interaction cross sections of $\bar{p}$'s (section 3.5) and parameters in Table 4.2, the equilibrium $\bar{p}$ flux is calculated in the same way as nuclei. This flux, however, includes only the secondary $\bar{p}$'s that are directly produced from the primary protons and nuclei. To obtain the flux of tertiary $\bar{p}$'s produced from the $\bar{p}$ non-annihilation interactions, the same calculation should be repeated by replacing the source term with the “tertiary source term,”

$$s_{\bar{p}-m}(E_{\bar{p}}) = \frac{\xi_{\bar{p}-\bar{p}}}{m_{\bar{p}}} \int_{E_{\bar{p}}}^{\infty} I_{\bar{p}-\bar{p}-m-1}(E'_{\bar{p}}) \frac{d\sigma_{\bar{p}p\rightarrow\bar{p}}}{dE_{\bar{p}}} dE'_{\bar{p}}$$

(4.26)

$$m = 1, 2, 3, ...$$

(4.27)

(see also eq. (3.12))

where the differential non-annihilation cross section of $\bar{p}$'s $d\sigma_{\bar{p}p\rightarrow\bar{p}}/dE_{\bar{p}}$ is given in section 3.6, Chapter 3. The first term $s_{\bar{p}-1}$ represents the source spectrum of once interacted $\bar{p}$'s, which is calculated from the secondary $\bar{p}$ flux $I_{\bar{p}-0}$. From this source term, the flux of once interacted $\bar{p}$'s $I_{\bar{p}-1}$ is obtained by the propagation calculation, and then the next source term $s_{\bar{p}-2}$ is obtained by eq. (4.26). In this way, we can obtain $I_{\bar{p}-m}$ in order of $m$, where $I_{\bar{p}-m}$ represents the flux of $\bar{p}$'s that have experienced $m$ times of non-annihilation interactions. Finally, the total $\bar{p}$ flux is given as

$$I_{\bar{p}} = \sum_{m=0}^{\infty} I_{\bar{p}-m}$$

(4.28)

The above summation was actually cut off at $m = 5$ in the present calculation. We have confirmed that the contribution above $m = 6$ is less than 1 % by performing the calculation up to $m = 50$. In the following arguments, we refer to the flux $I_{\bar{p}} - I_{\bar{p}-0}$ (i.e., the sum of $m \geq 1$) as tertiary $\bar{p}$ flux.

Fig. 4.10 shows the resultant IS $\bar{p}$ flux in the DR model. The results of three halo models are shown along with the secondary $\bar{p}$ flux under the $h_h = 2 \text{ kpc}$ model.
As shown, the tertiary $\bar{p}$’s in the DR model account for $\sim 20\%$ at 500 MeV, slightly smaller contribution compared with that in the SLB model: $\sim 30\%$. At lower energies, the difference of the fraction of tertiary $\bar{p}$’s between the DR and SLB models increases although such low-energy $\bar{p}$’s cannot be observed at 1 AU (Appendix B). This small fraction of tertiary $\bar{p}$’s in the DR model is due to the effect of the diffusion in energy space (energy-space diffusion) caused by the dispersion term of the Fermi acceleration (section 4.2). In the DR model, low-energy $\bar{p}$’s are produced by this energy-space diffusion [24], i.e., the last term in the left-hand side of eq. (4.15), and the contribution of tertiary $\bar{p}$’s in low-energy regions become relatively small compared with that in the SLB model, in which the $\bar{p}$’s below 500 MeV are mainly tertiary. Fig. 4.10 also shows that the $\bar{p}$ flux is not sensitive to the halo thickness $h_h$ like the flux of stable nuclei. In other words, we cannot obtain the information of the halo size from $\bar{p}$ flux.

Fig. 4.11 shows the interstellar $\bar{p}$ flux under the DR model (present result with $h_h = 2$ kpc and result of Simon et al. [4]) along with the result of the SLB model obtained in Chapter 3, namely, the thick solid line in Fig. 4.11 is the same as the thick solid line in Fig. 3.10, Chapter 3. As shown in Fig. 4.11, the $\bar{p}$ flux in the DR model is flatter than that in the SLB model. This is due to the energy-space diffusion effect caused by the energy dispersion in the collision $\langle (\Delta E)^2 \rangle_{\text{col}}$ (see Table 4.1). Since the antiproton source spectrum has a peak at $\sim 2$ GeV (Fig. 3.6), antiproton propagation is more sensitive to $\langle (\Delta E)^2 \rangle_{\text{col}}$ than $\langle \Delta E \rangle_{\text{col}}$, namely, the 2-GeV peak is broadened by the energy-space diffusion effect resulting in a flatter spectrum of the equilibrium flux. On the other hand, the nuclei flux in the DR model is dependent mainly on the energy gain $\langle \Delta E \rangle_{\text{col}}$ because of their source spectra steeply falling with energy. In this way, the effects of the Fermi acceleration on nuclei and $\bar{p}$ fluxes are quite different. This is the reason why the $\bar{p}$ fluxes in the SLB and DR models differ considerably although both models were tuned to fit the data of nuclei fluxes.

In such situation, the ratio of dispersion to energy gain $\langle (\Delta E)^2 \rangle_{\text{col}}/\langle \Delta E \rangle_{\text{col}}$ (Table 4.1) is very important for the $\bar{p}$ flux calculation. Simon et al. [4] used 2 times larger $\langle (\Delta E)^2 \rangle_{\text{col}}/\langle \Delta E \rangle_{\text{col}}$ ratio than ours resulting in flatter $\bar{p}$ flux as shown in Fig. 4.11 (thin dashed line). Although, at energies above 1 GeV, our result and that of Simon et al. agree, below 1 GeV, the flux of Simon et al. is considerably larger than ours. To confirm that this discrepancy is due to the discrepancy in the $\langle (\Delta E)^2 \rangle_{\text{col}}/\langle \Delta E \rangle_{\text{col}}$ ratio, we have performed a calculation using the $\langle (\Delta E)^2 \rangle_{\text{col}}/\langle \Delta E \rangle_{\text{col}}$ ratio under the 1-D scattering model, the result of which is shown by the thick dot-dashed line in Fig. 4.11 being in agreement with the result of Simon et al.. Here we cannot determine which is correct because both 1-D and 3-D scattering models are based on some simple assumptions concerning the scattering of cosmic rays by moving turbulences. Then, in principle, the $\langle (\Delta E)^2 \rangle_{\text{col}}/\langle \Delta E \rangle_{\text{col}}$ ratio should be determined experimentally, e.g., by measuring the antiproton spectrum precisely.

However, it should be noted that the 3-D scattering model is at least more realistic picture than 1-D model. In the 1-D model, one considers the head-on and overtaking collisions only. In the 3-D scattering model, the transverse scattering ($\cos \theta_{\text{rel}} \sim 0$) is also included. Since cosmic rays gain energy effectively with less
Figure 4.10: Interstellar $\bar{p}$ flux in the Diffusive Reacceleration model calculated with three models shown in Table 4.2. For comparison, the results of SLB model are also shown, which are identical with that shown in Fig. 3.10.
Figure 4.11: Antiproton flux in the SLB and DR models: the present results and works of Simon et al. [4]. Thick dashed line is the present result of the DR model ($h_h = 2$ kpc). Thick dot-dashed line was obtained using 1-D scattering model, where the dispersion of energy is twice larger than the present model (Table 4.1, see text).
Figure 4.12: Antiproton flux at TOA under the SLB and DR models. Solid and dashed lines respectively are the results of the SLB and DR models. The solar modulation calculation is the same as Fig. 3.12. Data points are from the BESS experiment [13] (filled squares), Bogolomov et al. [20, 21] (open triangles), and Golden et al. [22] (filled triangles).
Figure 4.13: $\bar{p}/p$ flux ratio at TOA under the SLB and DR models. The solar modulation calculation and interstellar proton flux are the same as Fig. 3.13. Data points are the same as Fig. 1.1, i.e., from the BESS experiment [13] (filled squares), IMAX experiment [14] (filled circles), PBAR experiment [16] (open diamonds), LEAP experiment [17, 18] (open circles), Buffington et al. [19] (open square), Bogolomov et al. [20, 21] (open triangles), and Golden et al. [22] (filled triangles).
dispersion in the transverse scattering, this scattering contributes to decreasing the 
\( \langle (\Delta E)^2 \rangle_{\text{col}} / \langle \Delta E \rangle_{\text{col}} \) ratio. We expect that this transverse scattering should occur in the interstellar space because the magnetic field is turbulent enough that the motion of cosmic rays can be treated as a random walk. The small value of the 
\( \langle (\Delta E)^2 \rangle_{\text{col}} / \langle \Delta E \rangle_{\text{col}} \) ratio in the 3-D scattering model is then more plausible than the twice larger one expected in the 1-D scattering model.

The solar modulation calculation was performed in the same way as Chapter 3. Fig. 4.12 shows the resultant \( \bar{p} \) flux at the top of the atmosphere (TOA) along with the data. As shown, the \( \bar{p} \) flux under the DR model (dashed lines) are smaller than that under the SLB model by a factor of \( \sim 2.5 \). The present data shows better agreement with the SLB model, although the DR model is also consistent with the data considering the errors of the calculation as well as the data. The error of SLB prediction is shown in Fig. 3.12 and that of DR prediction is expected to be larger considering the uncertainty in the degree of reacceleration \( \beta_B \). As other possibilities to explain the observed \( \bar{p} \) flux under the DR model, we can speculate on the following cases.

- The reacceleration is weaker, i.e., \( \beta_B \) is smaller. In this case, the model will not be consistent with the Kolmogorov turbulence of the magnetic field (\( \lambda_{\text{esc}} \propto R^{-1/3} \)).

- Novel primary \( \bar{p} \) source considerably contributes to the low energy flux.

Fig. 4.13 shows the \( \bar{p}/p \) flux ratio at TOA. To calculate this ratio, the same interstellar proton flux (eq. (3.1)) was used. This is valid because eq. (3.1) is directly obtained from the observation [58] and is not dependent on the propagation model. Of course, to examine whether this proton flux is consistent with the DR model is also of great importance. Such work was carried out by Seo and Ptuskin [42], and the observed proton flux was found to be consistent with the DR model.

As already explained in Chapter 3, the \( \bar{p}/p \) flux ratio is affected by the solar modulation especially at low energies. Although the difference between the SLB and DR models is larger than that between \( \phi = 700 \) and 600 MV (corresponding 1993 and 1994 BESS flight respectively), the test of the propagation model using this ratio is more difficult than that using \( \bar{p} \) absolute flux because we must monitor the solar activity level with great care in using \( \bar{p}/p \) flux ratio.
Chapter 5

Cosmic-Ray Positron Flux in the Diffusion and Diffusive Reacceleration Models

Positrons \((e^+)\) are also produced from high energy collisions in interstellar space, e.g., \(pp \rightarrow \pi^+ \rightarrow \mu^+ \rightarrow e^+\). From the fact that positron fraction at TOA \(e^+/(e^++e^-)\) is as small as \(\sim 0.1\), it is considered to be likely that positrons are mainly secondaries produced from the high energy collisions, while the electrons are mainly primaries accelerated at the source regions. However, the small positron fraction can also be explained by assuming the existence of the primary electron sources and the smaller contributions from primary \(e^\pm\) pair sources. We cannot determine whether the main component of positrons are the secondaries or not. However, we can calculate the secondary positron flux not being dependent on the problem of \(e^-\) and \(e^+\) primary sources because the secondary positrons are mainly produced from protons. By comparing the calculated secondary positron flux with the data, we will be able to obtain some informations on the origin of the cosmic-ray positrons and their propagation in the Galaxy.

If it becomes obvious that the positrons are mainly secondaries, they are useful for the probe of the cosmic-ray propagation. Positrons experience quite different processes from \(\bar{p}\)'s. First, because of their low mass (511 keV), positrons are relativistic even at energies as low as 100 MeV. Therefore the study of \(e^+\) flux is of great importance to investigate the propagation process related to the particle velocity. For example, we have obtained two parameterizations of the escape length in Chapter 2, i.e., \(\lambda_{esc}(R, \beta)\) and \(\lambda_{esc}(R)\), both of which fit the data of nuclei fluxes. However, in this Chapter, it will be shown that \(\lambda_{esc}(R, \beta)\) fits the \(e^+\) data, but \(\lambda_{esc}(R)\) does not fit them.

Second, high energy positrons lose energy by synchrotron radiation and inverse Compton scattering, which are expected to occur in the galactic halo as well as in the galactic disk. Then \(e^+\) flux is sensitive to the size of the galactic halo like radioactive nuclei such as \(^{10}\)Be. It is therefore of importance to test whether \(e^+\) flux and the \(^{10}\)Be/\(^9\)Be flux ratio can be fitted by the same model of the galactic halo.

The energy loss processes at high energies are important also for the test of the DR model because, in the DR model, the energy loss and acceleration processes
compete with each other resulting in some characteristic spectrum as shown in section 5.3.

5.1 Interstellar Source Spectrum of Positrons

To obtain the spectrum of the production rate of positrons ($e^+$ source spectrum) in interstellar space, we essentially take the same procedure as antiprotons (Chapter 3). First, the interstellar proton flux of eq. (3.1) is used.

To obtain the inclusive production cross sections of positrons $\sigma_{pp \rightarrow e^+X}$, we utilized the Monte Carlo code GEAN3.21 [83]. In our cross section calculation program, protons with a given energy are injected into the liquid hydrogen target of thickness $7 \text{ gcm}^{-2} (0.07 \text{ gcm}^{-3} \times 100 \text{ cm})$. Using the GHEISHA cross sections, the production of pions, kaons, and other hadrons is simulated. Lepton production is also included. Positron yields from the decay of these particles as well as the direct production are recorded for each small bin of the energy of produced positrons. The inclusive differential cross sections $d\sigma_{pp \rightarrow e^+X}/dE_{e^+}$ are calculated from this $e^+$ yields and the thickness of the target. Because the target is thin enough, the multiple interaction hardly occurs in the target, so that the inclusive cross sections are directly obtained from the $e^+$ yields.

By convolving the proton flux with positron production cross sections, we obtain $e^+$ source spectrum in interstellar space (Fig. 5.2 and 5.1). The result of Protheroe [5] is also shown by the dashed lines, which are in reasonable agreement with the present result. Unlike the source spectrum of antiprotons (dot-dashed line, Fig. 5.2),

![Figure 5.1: Positron source spectrum multiplied by cube of energy. Solid and dashed lines are the same as Fig. 5.2.](image-url)
Chapter 5. Cosmic-Ray Positron Flux in the Diffusion and DR Models

Figure 5.2: Positron source spectrum in interstellar space. Solid line is the present result. Dashed lines are the work of Protheroe [5]. At low and high energies, Protheroe has estimated the error bounds (upper and lower dashed lines) due to uncertainties in the positron production cross sections. Dot-dashed line is $\bar{p}$ source spectrum, which is identical with the thick solid line of Fig. 3.6.
the positron source spectrum does not have the peak. The reason is that positrons are mainly produced from the decay of pions, i.e., \( \pi^+ \rightarrow \mu^+ \rightarrow e^+ \), whose kinetic energy threshold is \( \sim 0.3 \) GeV; much lower than that of antiproton production (5.6 GeV), so that CMS is not boosted so much as the case of antiproton production. As shown in Fig. 5.1, the positron source spectrum is approximately a power-low spectrum with an exponent of \( \sim 2.7 \); the similar value as the exponent of the parent proton spectrum. Note that \( \bar{p} \) source spectrum also becomes a similar shape at high energies (dot-dashed line, Fig. 5.2), where the threshold effect is less.

Contributions from heavy nuclei in cosmic-ray beams and ISM targets are included by multiplying the \( \epsilon \) factor like the case of antiproton production (section 3.3, Chapter 3). When considered under the wounded nucleon model [1], the ratio of the inclusive particle production cross sections \( \sigma_{AB \rightarrow iX} / \sigma_{pp \rightarrow iX} \) is not dependent on the kind of produced particle \( i \), where \( A \) and \( B \) respectively are the nuclei in the beams and targets. This assumption results in the equal \( \epsilon \) value for the positron and antiproton productions. We thus take \( \epsilon = 1.16 \) according to eq. (3.11).

### 5.2 Calculation of \( e^+ \) Flux in the One-Dimensional Diffusion Model

Propagation of positrons is quite different from that of antiprotons and nuclei because of the energy loss by synchrotron radiation in the galactic magnetic field and inverse Compton scattering with the photons of star light and cosmic microwave background (CMB). This energy loss has quite different nature from the ionization energy loss as follows.

The ionization energy loss affects low energy particles only. For the antiprotons, the ionization energy loss is negligible above 100 MeV. The antiprotons below 100 MeV, however, cannot be observed at 1 AU due to the solar modulation effect. Therefore the ionization energy loss is not important for antiprotons. On the other hand, the energy loss rate \( (dE/dt) \) of the positrons by the synchrotron radiation and inverse Compton scattering is proportional to the square of energy, i.e.,

\[
\left\langle \frac{dE}{dt} \right\rangle^e = bE^2,
\]

where \( b \) is a constant and \( E \) the energy of the positrons. We use the \( b \) value estimated by Taira et al. [84];

\[
b = 1.97 \times 10^{-16} \text{ (GeV}^{-1}\text{s}^{-1}),
\]

which is based on the most plausible values of the intensity of the galactic magnetic field (r.m.s.), the energy density of star-light photons, and that of CMB photons.

This energy loss is more important for higher energy positrons. The positrons with high enough energy lose considerable energy before they escape from the Galaxy, so the their propagation is governed by this energy loss process rather than escape and diffusion processes. As they lose energy, however, the diffusion and escape processes become more and more important, while the energy loss becomes less
and less effective. In other words, the propagation in the energy space is important for high energy positrons, while the spatial propagation is important for low energy positrons. This transition should be treated with care. To do this, we have to give up the Leaky-Box approximation because this approximation does not consider the spatial diffusion. Details are as follows. The region where the positrons are produced is the galactic disk, while the energy loss occurs throughout the Galaxy. It is already explained in Chapter 4 that in such case the Leaky-Box approximation is not appropriate. The high energy positrons are produced in the galactic disk, and they lose considerable energy before they diffuse into the halo. So, the existence of the galactic halo is not effective in this phase. After their energy have reduced enough that energy loss is not effective very much, they can diffuse into larger halo, so that the halo propagation becomes important. To treat these processes correctly, the 1-D Diffusion model is necessary [43]. From the above argument, it is expected that the solution of the 1-D Diffusion model corresponds to Leaky-Box solution with the confinement volume of the galactic disk only for high-energy positrons and the confinement volume involving the galactic halo for low-energy positrons. This relation was shown by Nishimura et al. [43] using Fourier series solution of the 1-D Diffusion model.

The present calculation has been carried out by employing that Fourier series solution, whose details are described in Appendix C. Fig. 5.3 shows the resultant interstellar $e^+$ flux. The solid and dashed lines respectively are obtained using $\lambda_{esc}(R, \beta)$ and $\lambda_{esc}(R)$. The $\lambda_{esc}$ value of positrons given by these two parameterizations were shown in Fig. 2.6, Chapter 2. As described there, the values of $\lambda_{esc}(R, \beta)$ and $\lambda_{esc}(R)$ are very different for low-energy positrons, although those of nuclei are almost equal (Fig. 2.3) with nuclei data being well fitted by both parameterizations as shown in Fig. 2.5. Because of this difference in $\lambda_{esc}$, the interstellar $e^+$ flux obtained using $\lambda_{esc}(R)$ is much smaller than that obtained using $\lambda_{esc}(R, \beta)$ at low energies (Fig. 5.3).

Positron flux is also dependent on the thickness of the halo, because they lose energy in the halo. The three values of the halo thickness are tested, i.e., $h_h = 1, 2,$ and $4 \text{ kpc}$, which are the values that fit the $^{10}\text{Be}/^{9}\text{Be}$ data (Fig. 2.7 and 2.9, Chapter 2). As shown in Fig. 5.3, the smaller halo results in larger $e^+$ flux because in a smaller halo the residence time of cosmic rays in the Galaxy is shorter thereby reducing the loss of high energy positrons.

Fig. 5.4 is the same as Fig. 5.3 multiplied by cube of energy. The dot-dashed line is the source spectrum which is identical with the solid line in Fig. 5.1. At high energies, the positron flux more steeply falls than source spectrum. This is due to the energy dependence of the escape length at high energies, i.e., $\lambda_{esc} \propto R^{-0.65}$, but not due to the synchrotron and inverse Compton energy losses. This latter effect will also make the positron spectrum steeper, though this will become effective above 100 GeV [86, 84]. Therefore, the kink in the positron spectrum due to this effect has not yet been observed, although the position of this kink is sensitive to the halo thickness [86, 84]. At energies below 100 GeV where the observations are available, only the absolute value of the positron flux is sensitive to the halo thickness as shown in Fig. 5.3 and 5.4. Then the accuracy of the parent proton flux and other input
Figure 5.3: Interstellar positron flux calculated under the Diffusion model. Solid and dashed lines respectively are obtained using $\lambda_{\text{esc}}(R, \beta)$ and $\lambda_{\text{esc}}(R)$, with the halo thickness $h_h = 1, 2, 4$ kpc from above. Dot-dashed line is the source spectrum that is identical with the solid line in Fig. 5.2. Dotted line is the interstellar $e^++e^-$ flux based on the observations [85, 86, 87].
data is very important like the antiproton case.

To compare the results with the data, the solar modulation calculations are performed in the same way as antiprotons (Chapter 3, Appendix B). The solar activity level for the HEAT and other experiments are obtained from the CLIMAX neutron monitor data [60] (see Chapter 3). Thus modulated $e^+$ flux at TOA is divided by equally modulated ($e^+ + e^-$) flux, where the interstellar $e^+ + e^-$ flux (dotted lines, Fig. 5.3 and 5.4) is based on the observations [85, 86, 87].

Fig. 5.5 and 5.6 respectively show the resultant positron fraction at TOA calculated with $\lambda_{esc}(R, \beta)$ and $\lambda_{esc}(R)$. As shown, both results are in good agreement with the recent observations by the HEAT experiment [30]. It is important that this good agreement is obtained in the range of the halo thickness $h_h = 1$ to $4$ kpc, which is the same range as that constrained by the $^{10}\text{Be}/^{9}\text{Be}$ data (Chapter 2). This indicates that the main component of the cosmic-ray positrons is the secondary and they experience the same propagation process as the nuclei component involving the galactic halo with a half thickness of 1 to 4 kpc.

From the above arguments, it is natural to expect that the positrons below 1 GeV are also mainly secondaries because the primary sources of $e^\pm$ pair, if existed, are expected to have rather hard spectra [8] [9] not contributing only at low energies. Then the positron data and the present calculation should agree also at low energies. As shown in Fig. 5.5 and 5.6, only the results of $\lambda_{esc}(R, \beta)$ are in good agreement with the data, but the results of $\lambda_{esc}(R)$ do not fit the data. Therefore, we conclude that $\lambda_{esc}(R, \beta)$ is the realistic form of the escape length. This indicates that the cosmic-ray propagation in the Galaxy is not governed only by the particle rigidity but the particle velocity is also related.
Figure 5.5: Positron fraction at TOA calculated under the Diffusion model with $\lambda_{\text{esc}}(R, \beta)$. The interstellar flux (solid lines, Fig. 5.3) is modulated using spherically symmetric diffusion-convection model (Fisk’s numerical solution, see Appendix B), and divided by the $e^+ + e^-$ flux at TOA, which is obtained by the same modulation calculation using the interstellar $e^+ + e^-$ flux (dotted line, Fig. 5.3). Data points are from Fanselow et al. [25], Daugherty et al. [26], Buffington et al. [27], Muller et al. [28], Golden et al. [29], HEAT experiment [30], and AESOP experiment [31].
Figure 5.6: Positron fraction at TOA calculated under the Diffusion model with $\lambda_{esc}(R)$. The method of calculation is the same as Fig. 5.5 using the interstellar positron flux of dashed lines in Fig. 5.3. Data points are also the same as Fig. 5.5.
5.3 Calculation of $e^+$ Flux in the Diffusive Reacceleration Model

For the calculation of positron flux in the DR model, we use the same numerical technique as is used in the $\bar{p}$ flux calculation (Chapter 4). In Appendix C, the method of $\bar{p}$ and $e^+$ flux calculation is explained together. The positron source spectrum is the same as that shown in the previous section.

The resultant interstellar positron flux is shown in Fig. 5.8 and 5.7 along with the result of the Diffusion model with $\lambda_{esc}(R, \beta)$. As shown in these figures, $e^+$ flux at high energy is dependent only on $h_h$, being independent of the models (Diffusion or DR). As already shown in Chapter 4, $^{10}$Be/$^9$Be flux ratio is also dependent only on $h_h$. Therefore, we can determine the halo thickness $h_h$ from the data of high-energy $e^+$ flux and $^{10}$Be/$^9$Be flux ratio even if it is not known which of Diffusion and DR model is correct picture of the cosmic ray propagation.

On the other hand, $e^+$ flux around 2 GeV is sensitive to the reacceleration effect, i.e., $e^+$ flux around this energy is considerably larger than that in the Diffusion model. The reason is that positrons below this energy are effectively accelerated by Fermi acceleration, while those above this energy experience strong deceleration by the synchrotron radiation and inverse Compton scattering. From these two effects, positrons are gathered around 2 GeV where the acceleration and deceleration are in equilibrium.

Fig. 5.9 shows the positron fraction at TOA along with the data. The solar modulation calculation and $e^+ + e^-$ absolute flux are the same as the previous section. The $e^+$ fraction in the DR model steeply increases with decreasing energy.
Figure 5.8: Interstellar positron flux calculated under the Diffusion (solid lines) and DR (dashed lines) models with the halo thickness $h_h = 1, 2, 4$ kpc from above. Dotted line is the interstellar $e^+ - e^-$ flux based on the observations [85, 86, 87].
Figure 5.9: Positron fraction at TOA calculated under the DR model. The method of calculation is the same as Fig. 5.5 using the interstellar positron flux of dashed lines in Fig. 5.8. Data points are also the same as Fig. 5.5.
below $\sim 10$ GeV, which corresponds to the 2-GeV bump in the IS flux. As shown in the figure, the recent data of HEAT and AESOP experiments are consistent with this steep increase, while the data of Fanselow et al. [25] and those of Daugherty et al. [26] do not show such a steep increase. This discrepancy may be due to the charge asymmetric solar modulation effect [122, 121, 123, 124]. Further understanding of the solar modulation effect is necessary to draw conclusions.

In this Chapter, we have shown the following.

- The data of the HEAT experiment [30] (positron flux above several GeV) and the $^{10}\text{Be}/^9\text{Be}$ data (Chapter 2) are both explained by the Diffusion model with the halo thickness $h_h = 1$ to 4 kpc (Fig. 5.5).

- The data of positron flux below 1 GeV are in agreement with the prediction of the Diffusion model with the escape length $\lambda_{esc}(R, \beta)$ (Fig. 5.5), but do not agree with the results of $\lambda_{esc}(R)$ (Fig. 5.6).

- The positron flux below several GeV is sensitive to the reacceleration effect (Fig. 5.9).
Chapter 6

Local Flux of Low-Energy Antiprotons from Evaporating Primordial Black Holes

From a standpoint of cosmological understanding, it would be of great value to confirm the existence or nonexistence of primordial black holes (PBHs), which may have formed in the early Universe via initial density fluctuations, phase transitions, or the collapse of cosmic strings (for a review, see Ref. [88]). Hawking [89] first showed that black holes (BHs) emit particles and evaporate by quantum effects, noting that PBHs are the only ones with a mass small enough for the quantum emission rate to be significant, possibly yielding an observable effect. For example, the hard $\gamma$-rays from small enough PBHs may contribute to the diffuse $\gamma$-ray background spectrum, though no distinct signature has been observed. Thus, this leads to an upper limit (U.L.) on the average density of PBHs in the Universe, i.e., the ratio of their density to the critical density of the Universe, $\Omega_{\text{PBH}}$, must be $\lesssim 10^{-8}$ [90].

Despite such a stringent limit, their signature could still appear in the spectrum of cosmic-ray antiprotons ($\bar{p}$'s) [6]. This possibility arises because, although the kinematics of secondary $\bar{p}$ production should lead to a steep drop in the resultant $\bar{p}$ flux at kinetic energies less than 2 GeV as shown in Chapter 3 and 4, the expected flux of $\bar{p}$'s from PBHs (PBH-$\bar{p}$'s) has contrastingly been shown to increase with decreasing kinetic energy down to $\sim 0.2$ GeV [6]; thus providing a distinct signature below 1 GeV. Hence, searches for such low-energy cosmic-ray $\bar{p}$'s could lead to a novel constraint on the density of PBHs, or more importantly, demonstrate their existence. With this in mind, and spurred by the recent detection of cosmic-ray $\bar{p}$'s with kinetic energies less than 0.5 GeV [11, 12, 13], we present a new method describing the propagation of $\bar{p}$'s in the Galaxy based on the 3-D Diffusion model; thereby obtaining the most accurate-to-date spectrum of local interstellar PBH-$\bar{p}$ flux. We also use these results with observed data [13] to derive a new U.L. on the density of PBHs which are expiring near the Solar system, after which we discuss its cosmological aspects assuming PBHs to have formed via initial density fluctuations.
6.1 The Source Spectrum of PBH-\(\bar{p}\)’s

The source spectrum of PBH-\(\bar{p}\)’s is determined using general properties of BH evaporation [91, 92]. Briefly, an uncharged, non-rotating BH with mass \(M\) emits particles with spin \(s\) and total energy between \((Q, Q + dQ)\) at a rate [89]

\[
\frac{dN}{dt} = \frac{\Gamma_s dQ}{2\pi \hbar} \left[ \exp \left( \frac{Q}{kT} \right) - (-1)^{2s} \right]^{-1}
\]  

(6.1)

per degree of particle freedom, where \(T\) is the BH temperature (= \(h^2/8\pi G M k = 1.06 \times 10^{13}(M/g)^{-1}\) GeV, with \(k = 1\)), and \(\Gamma_s\) is the dimensionless absorption probability for the emitted species. Considering all species, the corresponding mass loss rate can be expressed as [92]

\[
\frac{dM}{dt} = -5.34 \times 10^{25} f(M) \left( \frac{M}{g} \right)^{-2} \text{ g s}^{-1},
\]  

(6.2)

where \(f(M)\), a function of the number of emitted species, is normalized to unity for large \(M\) (\(\geq 10^{17}\) g) and increases with decreasing \(M\). From Eq. (6.2), it follows that \(M_s \simeq 5.3 \times 10^{14}(t_u/16 \text{ Gyr})^{1/3} \text{ g}\), or \(3 \times 10^{-19}(t_u/16 \text{ Gyr})^{1/3}\) in units of the solar mass, where \(M_s\) is the initial mass of a PBH expiring today; i.e., its initial lifetime equals the present age of the Universe \(t_u\), being taken here as 16 Gyr [93]. PBHs with initial mass \(M_i < M_s\) should have completely evaporated by now, while those with \(M_i\) slightly larger than \(M_s\) have an extremely high present temperature and will soon expire by “explosion.”

Under the assumptions that (i) Eq. (6.1) holds for each emitted species of quarks and gluons and (ii) all PBH-\(\bar{p}\)’s are their fragments, which is consistent with observed \(e^+e^-\) annihilations, we calculate the PBH-\(\bar{p}\) source spectrum per unit volume via the following three-step procedure: (1) The JETSET 7.4 [94] Monte Carlo simulation code is used to obtain the fragmentation function \(d\Phi^{(j)}(E_p, Q)/dE_p\) describing the fragmentation of each emitted species \(j\) with total energy \(Q\) into \(\bar{p}\)’s with total energy \(E_p\); (2) The \(\bar{p}\) emission spectrum from a BH at present temperature \(T\), \(d\Phi_{\bar{p}}(E_p, T)/dE_p\), is calculated by convolving Eq. (6.1) with \(d\Phi^{(j)}(E_p, Q)/dE_p\) for \(Q \geq E_p\) and summing over all \(j\) and their degrees of freedom; and (3) The expected PBH-\(\bar{p}\) source spectrum per unit volume is obtained by convolving the PBH present temperature distribution \(dn/dT\) with \(d\Phi_{\bar{p}}(E_p, T)/dE_p\) for \(T \geq 0.1\) GeV, where \(n\) is the number of PBHs per unit volume. Under Eq. (6.2), \(dn/dT\) versus \(T\) (\(\geq 0.1\) GeV) is roughly a power-law function (\(\propto T^{-4.2}\)), with its normalization being solely determined by the value of the initial mass spectrum \(dn/dM_i\) at \(M_i = M_s\).

We checked the reliability of our use of JETSET 7.4 [94] via data from \(e^+e^-\) colliders, i.e., observed \(\bar{p}\) spectra obtained from the fragmentation of quarks and gluons at various \(Q\) relevant to evaporating PBHs. First, we verified that the predominant contribution to PBH-\(\bar{p}\)’s occurs at \(Q = 1–5\) GeV, after which the simulated \(\bar{p}\) spectra from quark fragmentation were compared with DASP data at \(Q = 1.8–2.5\) GeV [95] and ARGUS data at \(Q = 4.99\) GeV [95], while those from gluon fragmentation with ARGUS data from the direct decay of \(\Upsilon(1S)\), which mainly proceeds through...
three gluons with an average energy of $m_{\Upsilon(1S)}/3 = 3.2$ GeV [95]. Results showed the JETSET simulations to be in good agreement with all above data (not shown).

### 6.2 Propagation of PBH-$\bar{p}$’s using 3-D Diffusion Model

The propagation of PBH-$\bar{p}$’s in the Galaxy is described based on the Diffusion model since this model has been shown to explain most existing cosmic-ray data (see previous chapters). Although we have used the 1-D approximation for positrons and radioactive nuclei ($^{10}$Be), and the Leaky-Box approximation for stable nuclei and antiprotons, here we must use the 3-D Diffusion model because the PBHs are expected to be distributed widely in the Galactic halo. Since the 3-D Diffusion model is difficult to treat analytically or numerically, we calculate the local interstellar PBH-$\bar{p}$ flux utilizing a Monte Carlo simulation code; an expansion of the 1-D simulation by Owens and Jokipii [44].

We assume that the PBH spatial distribution is proportional to the mass density distribution within the Galactic halo, i.e., $\propto (1+(r/r_c)^2)^{-1}$ [96], where $r$ is a distance from the Galactic center and $r_c$ is the core radius of 7.8 kpc. As shown later, the local PBH-$\bar{p}$ flux arises only from nearly expired PBHs existing within a few kpc away from the Solar system, whose location is at $r = r_\odot = 8.5$ kpc. Accordingly, the local PBH-$\bar{p}$ flux can be calculated via the three-step procedure by simply using $dn(M_i = M_*, r = r_\odot)/dM_i$, which is parameterized by introducing an unknown parameter $\varepsilon_*$:

$$
\frac{dn}{dM_i}(M_i = M_*, r = r_\odot) = \varepsilon_* \frac{\rho_{h\odot}}{M_*^2},
$$

(6.3)

where $\rho_{h\odot}$ is the local density of halo dark matter ($\approx 0.3$ GeV cm$^{-3}$ [97]). Under Eq. (6.3), $\varepsilon_*$ represents the ratio of the density of PBHs with $M_i = M_*$ to $\rho_{h\odot}$.

Each $\bar{p}$ ejected from the fragmentation of quarks and gluons is assigned to an initial position $x_0$ and energy $E_0$, after which it travels $x$ to $x + u\sqrt{6D\Delta t} + (\nabla D)\Delta t$ in each subsequent time step $\Delta t$, where $u\sqrt{6D\Delta t}$ represents the effect of isotropic diffusion during $\Delta t$, $u$ is a unit vector with random direction, $D = D(x)$ is the diffusion coefficient, and $(\nabla D)\Delta t$ expresses the anisotropy of diffusion caused by the spatial gradient of $D$. We do not consider the effect of convection due to galactic wind, because Webber et al. [40] showed that it does not significantly affect cosmic-ray propagation. Regarding subsequent collisions and energy loss, the interstellar medium (ISM) is considered to consist of 90% hydrogen and 10% helium atoms [32], and its number density distribution is modeled as $n_{\text{atom}} = 1.1 \exp(-z/100\text{pc})$ atoms/cm$^3$, where $z$ is the perpendicular distance from the galactic plane ($z = 0$). While propagating through the ISM, $\bar{p}$’s lose energy by ionization and are lost by annihilation. Those $\bar{p}$’s passing near the Solar system ($< 25$ pc) are included in the flux calculation.

The Galaxy is modeled as a cylindrical diffusing halo [40] with a diameter of 40 kpc and halo thickness of $2h_h$. Free escape is assumed to occur at the boundaries. As normally done, we parameterize the diffusion coefficient as $D = D_0(z)D_1(R)$, where
Figure 6.1: (a) Distribution of $D_0(z)^{-1}$ in halo models I–III as shown in the $z$-direction where the Solar system is located on the galactic plane ($z = 0$). (b) Distribution of PBHs contributing to the local $\bar{p}$ flux $F$ near the Solar system with $\varepsilon_* = 1.0 \times 10^{-8}$ as shown in the $z$-direction. (c) The same as (b) shown in the $r$-direction where the Solar system is located at $r = 8.5$ kpc.
Figure 6.2: Local interstellar PBH-$\bar{p}$ flux calculated using model III (see Fig. 6.1 and text) with two possible values of $R$, i.e., $1 \times 10^{-2}$ and $1 \times 10^{-3}$ pc$^{-3}$yr$^{-1}$ (see text). Also shown is the secondary $\bar{p}$ flux calculated using the SLB model (Chapter 3) and the DR model (Chapter 4).
$R$ is the rigidity of the particle. We apply three halo models for $D_0(z)$ (Fig. 6.1(a) shows $D_0(z)^{-1}$): two with constant $D_0$ but different values of $h_h$ (models I and II), and one in which $D_0$ is dependent on $z$ (model III). Assuming $D_0$ is constant, $D_0/h_h$ and $D_1(R)$ are determined [40] as $(8 \pm 1.6) \times 10^5$ cm s$^{-1}$ and $\sim (R/GV)^{0.6}$, respectively, which fits the secondary to primary ratios of cosmic-ray nuclei, e.g., the ratios of boron to carbon (B/C) and sub-iron to iron (sub-Fe/Fe). These values are consistent with the escape length obtained in Chapter 2 considering the relation of the escape length and the diffusion coefficient (Appendix C). The value of $h_h$ is also constrained from 2 to 4 kpc [40] in order to fit the radioactive secondary to stable secondary ratios, e.g., the ratios of radioactive beryllium to stable beryllium ($^{10}$Be/$^9$Be), being consistent with the range constrained by our analyses (Chapter 2 and 5). Thus, $h_h = 2$ and 4 kpc in models I and II, respectively. Model III also fits the above ratios at the same $D_1(R)$.

The simulated distribution of PBHs contributing to the integrated $\bar{p}$ flux $F$ near the Solar system with $\varepsilon_{\star} = 1.0 \times 10^{-8}$ is shown for halo models I–III in Fig. 6.1(b) and (c) in the $z$- and $r$-direction, respectively. Note that all models show the same contribution at $z = 0$ (galactic plane) as they are constrained to reproduce the observed secondary to primary ratios which originate from secondary production occurring there. In addition, only PBHs within a few kpc away from the Solar system contribute a substantial flux. When considering these results along with the fact that the mass density within this region is relatively constrained [97], this indicates that the Solar system $\bar{p}$ flux is only slightly dependent on the assumed mass density distribution within the Galactic halo [96]. The $\bar{p}$ mean confinement time is also calculated by the code as 4.0, 7.9, and $8.0 \times 10^7$ yr for models I–III resulting in $F = 0.78, 1.53$ and $1.55 \times 10^{-1}$ m$^{-2}$s$^{-1}$sr$^{-1}$, respectively.

As we have shown that the local PBH-$\bar{p}$ flux can only be due to contributions from PBHs that are close to explosion and exist within a few kpc away from the Solar system, data from searches in which low-energy cosmic-ray $\bar{p}$'s are detected can directly constrain (or possibly reveal) the PBH explosion rate averaged over this local region, termed here as the local PBH explosion rate $\mathcal{R}$, i.e.,

$$\mathcal{R} \equiv \frac{dn}{d\tau_i}(\tau_i = t_u, r = r_\odot) = \frac{dn}{dM_i}(M_i = M_\star, r = r_\odot) \times \frac{M_\star}{3t_u},$$

(6.4)

where $\tau_i (= t_u(M_i/M_\star)^3$ at $M_i \approx M_\star$) is initial lifetime of PBHs with initial mass $M_i$. Fig. 6.2 shows simulated local interstellar PBH-$\bar{p}$ flux using model III with two possible values of $\mathcal{R}$, i.e., $1 \times 10^{-2}$ and $1 \times 10^{-3}$ pc$^{-3}$yr$^{-1}$.

6.3 Results and Discussion

Then, to compare this $\bar{p}$ flux with observational data, we converted it into the flux at the top of the atmosphere (TOA) using the same solar modulation model as is used in the previous chapters, i.e., the spherically symmetric diffusion-convection
model (Appendix B). Dividing it by the proton flux at TOA, which is calculated from the interstellar proton flux eq. (3.1) and the same modulation model, we also obtain the $\bar{p}/p$ flux ratio at TOA. Fig. 6.3 and 6.4 respectively compare the resultant $\bar{p}$ flux and $\bar{p}/p$ flux ratio at TOA for PBH-\$\bar{p}$-s with the expected flux for secondary $\bar{p}$'s under the SLB and DR models (see Chapter 3 and 4 respectively) and observational data. Note that no distinct signature of evaporating PBHs is apparent, i.e., at kinetic energies below 1 GeV, the data shows no tendency to reach a constant $\bar{p}/p$ flux ratio. As no signature exists, statistical analysis of recent observations [13] leads to the following U.L. on $R$ with 90% confidence level (C.L.):

$$R < 1.3 \times 10^{-2} \text{ pc}^{-3}\text{yr}^{-1},$$

which is almost eight orders of magnitude more stringent than the present U.L. on the rate of 50-TeV $\gamma$-ray bursts ($R < 8.5 \times 10^5 \text{ pc}^{-3}\text{yr}^{-1}$ [98]), and the practical sensitivity for 100-MeV $\gamma$-ray bursts ($R \sim 10^6 \text{ pc}^{-3}\text{yr}^{-1}$ [99]). Note that, although the observed diffuse $\gamma$-ray background spectrum places an upper limit on the average density of PBHs in the Universe, it cannot be used to set a direct limit on the local $R$ because of the uncertainties on PBH clustering [90].

Equations (6.3)–(6.5) give the following U.L. on $\varepsilon_*$ with 90% C.L.:

$$\varepsilon_* < 2.2 \times 10^{-8} \left( \frac{\rho_h \odot}{0.3 \text{ GeV cm}^{-3}} \right)^{-1} \left( \frac{t_u}{16 \text{ Gyr}} \right)^{4/3},$$

which can be used to derive an U.L. on the fraction of the Universe’s mass going into PBHs with mass $M_*$, i.e., $\beta(M_*)$ [88]. If PBHs are assumed to have formed via initial density fluctuations, $\beta(M_*)$ should be closely related to their amplitude on a scale of $M_*$. Thus, under this assumption, and for a flat Friedmann universe, Eq. (6.3) gives $\beta(M_*) \sim 1 \times 10^{-18} \varepsilon_* (\Omega_h/0.1)$, where $\Omega_h$ is the ratio of the density of halo dark matter to the critical density of the Universe. Finally, assuming $\Omega_h = 0.1$ [93], Eq. (6.6) leads to an U.L. of $\beta(M_*) < 2 \times 10^{-26}$.

Further, by assuming that such initial density fluctuations are scale-invariant, the initial mass spectrum of PBHs should have a power-law form, i.e., $dn/dM_i \propto M_i^{-\alpha}$ [88], where $\alpha = 5/2$ if the Universe was radiation-dominated when PBHs formed. Using this initial mass spectrum normalized by Eq. (6.3) at $M_i = M_*$, we integrate $M_i \times dn/dM_i$ over $M_* \leq M_i \leq \infty$ to obtain $\Omega_{\text{PBH}} \equiv \varepsilon_* \Omega_h \int_1^\infty x^{1-\alpha} dx = 2 \times 10^{-1} \varepsilon_* (\Omega_h/0.1)$, where $x \equiv M_i/M_*$. Finally, assuming $\Omega_h = 0.1$ [93], Eq. (6.6) leads to an U.L. of $\Omega_{\text{PBH}} < 5 \times 10^{-9}$, being comparable to that from the diffuse $\gamma$-ray background spectrum ($<10^{-8}$) [90].

In closing, if future long-duration ($\sim 8$ days) balloon flights allow us to precisely measure the cosmic-ray $\bar{p}$ flux at kinetic energies from 0.2 to 2 GeV, the U.L. on $R$ can be significantly reduced to $\sim 2 \times 10^{-3} \text{ pc}^{-3}\text{yr}^{-1}$, or more importantly, such observations could confirm the existence of evaporating PBHs.
Figure 6.3: PBH-\bar{p} flux at TOA, modulated from the interstellar flux shown in Fig. 6.2 with \( \phi = 600 \text{ MV} \). Data points are from the BESS experiment [13] (filled squares), Bogolomov et al. [20, 21] (open triangles), and Golden et al. [22] (filled triangles).
Figure 6.4: $\bar{p}/p$ flux ratio at TOA for the PBH-$\bar{p}$'s (dot-dashed lines) along with the secondary $\bar{p}$ flux under the SLB and DR models. The solar modulation calculation and interstellar proton flux are the same as Fig. 3.13. Data points are the same as Fig. 1.1, i.e., from the BESS experiment [13] (filled squares), IMAX experiment [14] (filled circles), PBAR experiment [16] (open diamonds), LEAP experiment [17, 18] (open circles), Buffington et al. [19] (open square), Bogolomov et al. [20, 21] (open triangles), and Golden et al. [22] (filled triangles).
Chapter 7

Conclusions

We have calculated the flux of cosmic-ray antiprotons and positrons as well as the stable and radioactive nuclei in the Diffusion and Diffusive Reacceleration (DR) models. The Diffusion model assumed a diffusing galactic halo with thickness of a few kpc and a thin galactic disk (half-thickness 100 pc) that contains the cosmic-ray sources, ISM, and the Solar system. In the DR model, the Fermi acceleration during the propagation was also included.

For the shape of the confinement volume, the three types of approximation were used, i.e., 3-D halo model, 1-D halo model, and the Leaky-Box approximation, which are compiled in Table 7.1. The validity of the approximations was confirmed in Chapter 2. As shown there, the above approximations do not change the results significantly. It should be noted, therefore, that the flux calculations of various particles were carried out under the same physical picture. The Diffusion and DR models were thus tested by comparing the calculated and observed fluxes.

The results of the Diffusion model were as follows. First, the energy-dependent escape length \( \lambda_{esc} \) was tuned using the data of stable nuclei flux to be \( 17.0 \times (R/4.4 \text{ GV})^{-0.65} \) (gcm\(^{-2}\)) above 4.4 GV, where \( R \) is the particle rigidity (Chapter 2). The halo thickness \( h_h \) was also constrained using the data of radioactive

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Table 7.1: Approximations used in the present calculations.

<table>
<thead>
<tr>
<th></th>
<th>Diffusion model</th>
<th>DR model</th>
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<tbody>
<tr>
<td>3-D halo</td>
<td>PBH-( \bar{p} ) (Chapter 6)</td>
<td></td>
</tr>
<tr>
<td>1-D halo</td>
<td>(^{10})Be (Chapter 2)</td>
<td>B, C, ..., (^{10})Be, ( \bar{p} ) (Chapter 4)</td>
</tr>
<tr>
<td></td>
<td>( e^+ ) (Chapter 5)</td>
<td>( e^+ ) (Chapter 5)</td>
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<td></td>
<td>“1-D Diffusion model”</td>
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<tr>
<td>Leaky-Box</td>
<td>B, C, ... (Chapter 2)</td>
<td></td>
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<td></td>
<td>( \bar{p} ) (Chapter 3)</td>
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<td></td>
<td>“SLB model”</td>
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</table>

† The DR model with 1-D halo is not called “1-D DR model” to avoid confusing with the 1-D scattering model. Note that we used the 3-D scattering model as explained in Chapter 4.
nuclei $^{10}$Be to be between 1 and 4 kpc (Chapter 2). Using thus obtained parameters, antiproton and positron fluxes were calculated and the results were compared with the data of the BESS experiment [13], HEAT experiment [30], and other data. As a result, we have found the following.

- The low energy antiproton data of the BESS experiment [13] is in good agreement with the prediction of the Diffusion model (SLB model, Chapter 3).

- The high energy positron data of the HEAT experiment [30] are also in good agreement with the prediction of the Diffusion model (Chapter 5).

- The range of the halo thickness constrained by these positron data is $h_h = 1$ to 4 kpc, which is in good agreement with the range constrained by the $^{10}$Be data.

- Low energy positron data agree with the prediction of the Diffusion model, if we use the parameterization of the escape length as a function of particle rigidity and velocity, i.e., $\lambda_{esc}(R, \beta)$. On the other hand, the escape length parameterized as a function of rigidity only, $\lambda_{esc}(R)$, did not fit the low-energy positron data.

From these results, the Diffusion model was found to well explain the antiproton and positron fluxes as well as the fluxes of the stable and radioactive nuclei. Furthermore, it was found that the main component of the cosmic-ray antiprotons and positrons are probably the secondaries from the high energy interactions in interstellar space, such as $pp$ collisions mainly. This further indicates that the propagation of the dominant proton component can also be explained under the Diffusion model. It was also indicated by the positron results that the propagation of low-energy particle is related to the particle velocity as well as the rigidity.

The results of the DR model are as follows. Like the analyses of the Diffusion model, the escape length was tuned using the data of the stable nuclei flux to be $(10.3 \sim 11.2) \times (R/GV)^{-1/3} \text{ (gcm}^{-2}\text{)},$ which agrees with the previous results [24, 42]. The halo thickness was constrained using the $^{10}$Be data to be between 1 and 4 kpc, which is the same range as that under the Diffusion model. The strength of reacceleration $\beta_B C$ (the speed of the turbulent magnetic field line) should be $\sim 60$ km/s to fit these data with a single power-law escape length. Using thus obtained parameters, antiproton and positron fluxes were calculated and the results were compared with the data. The results are as follows.

- The low energy antiproton flux calculated under the DR model is about 2.5 times smaller than that under the Diffusion model, resulting in worse agreement with the data of the BESS experiment (Chapter 4).

- The positron flux above several GeV calculated under the DR model is almost equal to that under the Diffusion model, then agrees with the data of the HEAT experiment (Chapter 5).

- The range of the halo thickness that fits the positron data is $h_h = 1$ to 4 kpc, which agrees with that constrained by the $^{10}$Be data, and is the same range as that under the Diffusion model.
The positron flux around 2 GeV has very different spectral shape between Diffusion and DR models. However, the positron flux in this energy range is much affected by the solar modulation effect, and conclusions are not permitted at present.

From these results, we have found that the low energy antiproton flux is the most sensitive to the difference of the models (Diffusion or DR). Although the present data of the BESS experiment is in better agreement with the prediction of the Diffusion model, the conclusion is not permitted considering the errors of calculations as well as the data. However, since the errors of the calculations are smaller than the difference between Diffusion and DR models, the future precise measurement of the low energy antiproton flux will be able to determine which model is better.

Local flux of low-energy antiprotons from evaporating primordial black holes (PBHs) was also calculated using general properties of black hole evaporation [91, 92] and 3-D Diffusion model for the propagation of the \( \bar{p} \)'s form PBHs (PBH-\( \bar{p} \)'s) in the Galaxy. The resultant PBH-\( \bar{p} \) flux at the top of the atmosphere increases with decreasing energy unlike the secondary \( \bar{p} \) flux, which decrease with decreasing energy below 2 GeV. As data shows no signature of the PBH-\( \bar{p} \)'s, we have obtained the following upper limit (U.L.) on the PBH explosion rate averaged over the local region within a few kpc away from the Solar system \( \mathcal{R} \) with 90% confidence level (C.L.):

\[
\mathcal{R} < 1.3 \times 10^{-2} \text{ pc}^{-3}\text{yr}^{-1},
\]  

(7.1)

which is almost eight orders of magnitude more stringent than the present U.L. on the rate of 50-TeV \( \gamma \)-ray bursts (\( \mathcal{R} < 8.5 \times 10^5 \text{ pc}^{-3}\text{yr}^{-1} \) [98]), and the practical sensitivity for 100-MeV \( \gamma \)-ray bursts (\( \mathcal{R} \sim 10^6 \text{ pc}^{-3}\text{yr}^{-1} \) [99]).
Acknowledgments

First, I am grateful to Professor S. Orito for suggesting this subject and supporting overall my works, also for the valuable suggestions about interpretation of the latest observations of the BESS experiment. Sincere gratitude is to Professor J. Nishimura for stimulating discussions and suggesting important ideas especially based on his own works on cosmic-ray propagation. I also thank Mr. K. Maki for allowing me to include the works on the evaporating primordial black holes into my dissertation, and for helpful discussions on the associated research.

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Appendix A

Atmospheric Antiproton Flux at Balloon Altitude

To confirm that the antiprotons observed by balloon experiments are certainly of cosmic origin, we calculate the flux of atmospheric antiprotons, which is produced by the interactions of cosmic-ray protons (and heavier nuclei) with the nuclei in the residual atmosphere (e.g., 5 g cm$^{-2}$ at a typical balloon altitude 36000 m). Unlike the previous calculations [100, 101, 102], we employ 3-D Monte Carlo method, which incorporated the exact distribution of upper atmosphere (based on the US standard atmosphere) and angular distributions of $\bar{p}$ production and interactions based on the differential cross sections shown in Chapter 3. Using this Monte Carlo code, we can also obtain the angular distribution of atmospheric antiprotons at a given altitude, which is important for the large acceptance experiment like BESS [11].

Moreover, we have developed a model to calculate the differential cross sections of $p^{14}\text{N} \rightarrow \bar{p}X$ and $p^{16}\text{O} \rightarrow \bar{p}X$ interactions based on the experimental data of $p^{9}\text{Be} \rightarrow \bar{p}X$, $p^{12}\text{C} \rightarrow \bar{p}X$, and $p^{27}\text{Al} \rightarrow \bar{p}X$. Although these interactions are essentially explained as those of proton and “nuclear nucleon” (nucleon in the nucleus), the Fermi momentum of nuclear nucleons plays an important role especially at low energies including “sub-threshold particle production” [103, 104, 109, 110]. Then, we convolve the $pp \rightarrow \bar{p}X$ cross sections with the Fermi momentum distribution of Shor et al. [103], which includes the high momentum component (HMC) [105] as well as the normal component [106].

Particle production with high transverse momentum ($P_t$) [107] is also important for the $\bar{p}$ production in the air. It is known that [107, 108] the inclusive cross section of high-$P_t$ $\bar{p}$ production has anomalously strong dependence on the target mass number $A$. When the $A$ dependence of the production cross section is written in power-low, i.e., $\propto A^\alpha$, the exponent $\alpha$ increases with increasing $P_t$. As a result, the high-$P_t$ $\bar{p}$ yield would be underestimated if assuming that production cross section is simply proportional to $A^{2/3}$. Then, we use the measured $\alpha$ as a function of $P_t$ [107].

In this way, we can calculate the differential cross sections of inclusive $\bar{p}$ production from various nuclei and can compare them with the data. The results of these comparisons are shown in Fig. A.1-A.7. As shown in these figures, the present model well reproduces the experimental data.
Figure A.1: Sub-threshold $\bar{p}$ production cross sections. Data points are from Lepikhin et al. [109]. The laboratory angle and kinetic energy of the detected $\bar{p}$'s respectively are $\theta^{LS}_{\bar{p}} = 10.77$ degree and $E_{\bar{p}} = 1.056$ GeV. Lines are the corresponding calculated values based on the present model.
Figure A.2: Sub-threshold $\bar{p}$ production cross sections. Data points are from Chiba et al. [110]. The laboratory angle of the detected $\bar{p}$’s is $\theta_{\bar{p}}^{LS} = 5.1$ degree. Lines are the corresponding calculated values based on the present model.
Figure A.3: $\bar{p}$ production cross sections at $E_p = 9.106$ GeV ($E_p$ is the kinetic energy of incident protons). Data points are from Vaisenberg et al. [115]. The laboratory angle of the detected $\bar{p}$'s is $\theta^L_{\bar{p}} = 10.77$ degree. Lines are the corresponding calculated values based on the present model (solid lines) and obtained by simply multiplying $\sigma_{pp \rightarrow \bar{p}}$ ([65], see Chapter 3) by $A^{2/3}$ (dashed lines).
Figure A.4: $\bar{p}$ production cross sections at $E_p = 9.205$ GeV ($E_p$ is the kinetic energy of incident protons). Data points are from Sibirtsev et al. [114]. The laboratory angle of the detected $\bar{p}$’s is $\theta^L_{\bar{p}} = 3.5$ degree. Lines are the corresponding calculated values based on the present model (solid lines) and obtained by simply multiplying $\sigma_{pp \rightarrow \bar{p}}$ ([65], see Chapter 3) by $A^{2/3}$ (dashed lines).
Figure A.5: $\bar{p}$ production cross sections at $E_p = 23.08$ and $25.08$ GeV ($E_p$ is the kinetic energy of incident protons). Data points are from Kalmus et al. [113]. The laboratory angle of the detected $\bar{p}$’s is $\theta_{LS}^{\bar{p}} = 17.78$ degree. Lines are the corresponding calculated values based on the present model (solid lines) and obtained by simply multiplying $\sigma_{pp \rightarrow \bar{p}}$ ([65], see Chapter 3) by $A^{2/3}$ (dashed lines).
Figure A.6: $\bar{p}$ production cross sections at $E_p = 69.1$ GeV ($E_p$ is the kinetic energy of incident protons). Data points are from Barkov et al. [111, 112]. The laboratory angle of the detected $\bar{p}$'s is $\theta_{\bar{p}}^{LS} = 0$ degree. Lines are the corresponding calculated values based on the present model (solid lines) and obtained by simply multiplying $\sigma_{pp \rightarrow \bar{p}}$ ([65], see Chapter 3) by $A^{2/3}$ (dashed lines)
Figure A.7: High-$P_t$ $\bar{p}$ production cross sections at $E_p = 69.1$ GeV ($E_p$ is the kinetic energy of incident protons). Data points are from Abramov et al. [107]. The laboratory angle of the detected $\bar{p}$’s is $\theta_{\bar{p}}^{LS} = 9.17$ degree (90 degree in CMS). Lines are the corresponding calculated values based on the present model (solid lines) and obtained by simply multiplying $\sigma_{pp \rightarrow \bar{p}}$ ([65], [108]) by $A^{2/3}$ (dashed lines).
Using these $\bar{p}$ production cross sections and primary proton flux at the top of the atmosphere (TOA) shown in Fig. 3.11, the atmospheric $\bar{p}$ flux is calculated with 3-D Monte Carlo method. The contribution of primary helium is also included by multiplying the $\bar{p}$ production rate from protons by a factor 1.2 [100]. The interaction and annihilation of $\bar{p}$’s are taken into account according to the cross sections obtained in Chapter 3.

Fig. A.8 shows the resultant atmospheric $\bar{p}$ flux at an altitude 36000 m (residual atmosphere of 5 gcm$^{-2}$). As shown by the solid and dashed histograms, the atmospheric $\bar{p}$ flux is not dependent on the solar activity level very much because they are mainly produced from the protons above 10 GeV, whose flux is almost unchanged. The result of Stephens (also at 5 gcm$^{-2}$) [100] is also shown by thin solid line. Stephens used the primary proton flux of Webber et al., which is 1.6 times larger than that used here (see Fig. 3.4). To correct for this discrepancy in the proton flux, the atmospheric $\bar{p}$ flux of Stephens is multiplied by a factor 1/1.6 (thin dot-dashed line). As a result, the discrepancy between the thick solid histogram and the thin dot-dashed line is considered to originate from the differences in the $\bar{p}$ production cross sections. Stephens also took Fermi momentum into account, but high momentum component (HMC) [105] was not included, resulting in smaller $\bar{p}$ flux than ours. Production of high-$P_t$ $\bar{p}$ [107] also contributes to increasing atmospheric $\bar{p}$ flux in our calculation. Even with all such effects included, the atmospheric $\bar{p}$ flux at a typical balloon altitude is smaller enough than the expected $\bar{p}$ flux at TOA (dotted line).
Figure A.8: Calculated atmospheric $\bar{p}$ flux at 5 gcm$^{-2}$ (36000 m). Thick histograms show the present results for $\phi = 550$ MV (solid line, corresponding to $\sim$1995) and $\phi = 1400$ MV (dashed line, solar maximum). Thin solid and dot-dashed lines respectively show the result of Stephens [100], and that multiplied by 1/1.6 for the correction for the primary proton flux (see text). Also shown is the $\bar{p}$ flux at TOA predicted by the SLB model (Chapter 3).
Appendix B

Solar Modulation of the Galactic Cosmic Rays

To describe the cosmic-ray propagation in the Solar system and explain the solar modulation, Gleeson and Axford [116] established the “spherically symmetric diffusion-convection model.” In this minimal model, all quantities, i.e., the speed of solar wind and the diffusion coefficient, are dependent only on the distance from the sun \( r \). In such a spherically symmetric region (called the “Heliosphere”), cosmic rays experience diffusion process due to the turbulent interplanetary magnetic field, which is similar to the propagation in the Galaxy but with much more short scattering mean free path (about \( 10^{-5} \) of that in the Galaxy). At the same time, they experience convection process due to the outward solar wind, and also experience the adiabatic deceleration because the solar wind is expanding.

The transport equation of cosmic rays in the Heliosphere is [116]

\[
\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 S) = -\frac{V}{3} \frac{\partial^2}{\partial r \partial E} (\alpha E N) \tag{B.1}
\]

\[
S = V N - \kappa \frac{\partial N}{\partial r} - \frac{V}{3} \frac{\partial}{\partial E} (\alpha E N), \tag{B.2}
\]

where \( V \) is the speed of solar wind, \( \kappa \) the diffusion coefficient, \( N \) the cosmic ray density, \( E \) the kinetic energy, and

\[
\alpha = \frac{\mathcal{E} + m}{\mathcal{E}}, \tag{B.3}
\]

\[
\mathcal{E} = E + m, \tag{B.4}
\]

where \( m \) is the mass of particle and \( \mathcal{E} \) the total energy.

Although the general analytic solution of the above equations does not exist, Gleeson and Axford obtained an approximate solution [117] by setting \( S = 0 \) and assuming that

\[
\kappa = \beta \kappa_1(r, t) \kappa_2(R, t), \tag{B.5}
\]

where \( t \) is time and \( R \) the particle rigidity. The approximate solution is [117]
\[
\frac{I(r, E, t)}{E^2 - m^2} = \frac{I(r_b, E_b)}{E_b^2 - m^2}, \tag{B.6}
\]
\[
\zeta(E_r, Z, t) + \phi = \zeta(E_b, Z, t), \tag{B.7}
\]

where \(I(r, E, t)\) is the differential flux at position \(r\) and time \(t\), which is related to the density as
\[
I(r, E, t) = \frac{\beta c}{4\pi} N(r, E, t), \tag{B.8}
\]
and \(I(r_b, E_b)\) is the flux at the boundary of the Heliosphere \(r = r_b\), which is taken to be equal to the interstellar flux. \(E_b\) is the kinetic energy at \(r_b\) and \(E_b = E_b + m\). The functions \(\zeta\) and \(\phi\) are defined as
\[
\phi(r, t) \equiv \int_r^{r_b(t)} \frac{V(r', t)}{3\kappa_1(r', t)} dr', \tag{B.9}
\]
\[
\zeta(E, Z, t) \equiv \int_0^E \frac{\kappa_2(R', t)}{\sqrt{E'^2 - m^2}} dE', \tag{B.10}
\]

where \(Z\) is the particle charge, and \(R'\) and \(E'\) in eq. (B.10) are the values corresponding to \(E'\) (\(E' = E_b + m\), etc.). The solution (B.6) corresponds to the Liouville theorem, which shows that in this approximation \(\phi\) is the only parameter to represent the degree of the solar activity.

Furthermore, by assuming that
\[
\kappa_2 = R, \tag{B.11}
\]
Eqs. (B.7) and (B.10) are reduced to
\[
E_b = E_r + Z\phi. \tag{B.12}
\]

The above equation means that a cosmic-ray particle lose an energy \(Z\phi\), which is independent of the initial energy as if the particle travels in the static field ("Force Field approximation") [117]. Note that the energy loss per charge \(\phi\) is usually written in [MV] but is not a rigidity.

Instead of eq. (B.11), Perko [120] assumed that
\[
\kappa_2 = R \quad (R > R_c), \tag{B.13}
\]
\[
\kappa_2 = R_c \quad (R < R_c), \tag{B.14}
\]
where \(R_c\) is a constant. In this case, eq. (B.10) is calculated to be
\[
\zeta(E, Z) = R_c \left( \frac{1}{2} \ln \frac{E + R_c Z}{E - R_c Z} + \frac{E - E_c}{Z} \right) \quad (R > R_c), \tag{B.15}
\]
\[
\zeta(E, Z) = R_c \left( \frac{1}{2} \ln \frac{E + R Z}{E - R Z} \right) \quad (R < R_c), \tag{B.16}
\]

where \(E_c = \sqrt{(R_c Z)^2 + m^2}\). From eq. (B.7), (B.15) and (B.16), the relation between the interstellar energy and energy at \(r\) is [120]
\[ E_b = E_r + Z\phi \quad (R > R_c), \quad (B.17) \]
\[ E_b = R_c Z \ln \frac{E_r + R_r Z}{E_c - R_c Z} + E_c + Z\phi \quad (R < R_c), \quad (B.18) \]

where \( E_r \) and \( R_r \) are the values corresponding to \( E_r \). Also in this model, cosmic rays that travel from \( r_b \) to \( r \) lose a given energy deterministically, but the amount of energy loss is dependent on the initial energy. From eq. (B.18), it is found that the energy loss in this model becomes smaller than that in the simple Force Field approximation at low energies [120].

Fisk [118] showed the numerical method to solve eqs. (B.1) and (B.2) without using the approximation that \( S = 0 \). From this numerical solution, it was shown that the energy loss of the cosmic rays in the Heliosphere is not actually determined uniquely but has a statistical width, then a monochromatic spectrum in interstellar space (if existed) results in broad energy spectrum at 1 AU [119]. Another difference between the Force Field and Fisk’s solutions is that Fisk’s solution is, in principle, dependent on the form of \( \kappa_1(r, t) \), which is associated with the structure of the interplanetary magnetic field, while the Force Field solution is dependent only on the integral of \( \kappa_1 \) from \( r \) to \( r_b \) (eq. (B.9)).

To examine these natures, we calculated 1-AU spectrum from a monochromatic interstellar proton (antiproton) spectrum (Fig. B.1) using the Force Field and Fisk’s numerical solutions. For the Fisk’s solution, we assumed constant \( \kappa_1 \) throughout the Heliosphere, i.e.,

\[ \phi = \frac{V(r_b - 1 \text{AU})}{3 \kappa_1}, \quad (B.19) \]

which was fixed at 800 MV (solar medium, see Fig. 3.2 and 3.3). \( V = 400 \text{ km/s} \) was also fixed, and we tried the two models, i.e., \( r_b = 30 \text{ AU} \) and 60 AU. The Force Field solution was also obtained with \( \phi = 800 \text{ MV} \). As shown in Fig. B.1, the 1-AU spectra calculated using Fisk’s solution have considerably large width especially at low energies, and are very different from the Force Field solutions. However, two Fisk’s solutions calculated with \( r_b = 30 \) and 60 AU are in good agreement within the width of lines. Therefore, we concluded that the form of \( \kappa_1 \) is not effective to the 1-AU flux, but the Force Field approximation is not always appropriate.

The difference between the Force Field and Fisk’s solutions is, however, much reduced by using the realistic interstellar spectrum, such as rigidity power-low proton spectrum and SLB antiproton spectrum, as shown in Fig. 3.11. The differences are further smaller for the flux ratios of nuclei because the energy spectra of nuclei fluxes are all similar. We used, therefore, the Force Field solution for the nuclei flux ratios (Chapter 2) and Fisk’s numerical solution with \( r_b = 60 \text{ AU} \) for protons, antiprotons (Chapter 3, 4, and 6), and electrons/positrons (Chapter 5).
Figure B.1: 1-AU flux modulated from the monochromatic interstellar proton (antiproton) spectrum (solid lines) using the Force Field approximation (dotted lines) and Fisk’s numerical solution (dashed lines). Two dashed lines in each figure show the results of $r_b = 30$ and 60 AU, which agree within the width of lines.
Appendix C

Solutions of the One-Dimensional Diffusion and DR Models

All the calculations of the one-dimensional (1-D) Diffusion model (Chapter 2 and 5) have been performed using the following Fourier series solutions. For the nature of these solutions, one can refer e.g., [10] and [43]. The numerical technique to solve the transport equations of the DR model with 1-D halo (Chapter 4 and 5) is also shown here.

The transport equation of the 1-D Diffusion model was shown in Chapter 2, i.e.,

\[
\frac{\partial N_i}{\partial t} - D \frac{\partial^2 N_i}{\partial z^2} + \left( \frac{1}{\tau_i} + \frac{1}{\gamma \tau_{dec}} \right) N_i + \frac{\partial}{\partial E_i} \left( \frac{dE}{dt} N_i \right) = Q_i + S_i, \tag{C.1}
\]

(the same as eq. (2.3))

where \( z \) is the perpendicular distance from the galactic plane (\( z = 0 \)). We consider the steady-state solution, i.e.,

\[
\frac{\partial N_i}{\partial t} = 0, \tag{C.2}
\]

and assume that, at the boundary of the halo, cosmic-ray densities become zero, i.e.,

\[
N_i(z = \pm h_h) = 0, \tag{C.3}
\]

where \( h_h \) is the half thickness of the halo.

Under these conditions, the solution of eq. (C.1) is obtained as a Fourier series [10, 43],

\[
N_i(E_i, z) = \sum_{n=0}^{\infty} N_i^n(E_i) \cos \left( \frac{2n + 1}{2} \frac{\pi z}{h_h} \right). \tag{C.4}
\]

By inserting eq. (C.4) and (C.2) into (C.1), the equation for each Fourier coefficient is obtained as
\[ D \left( \frac{2n + 1 \pi}{2 h_h} \right)^2 N_i^n + G_{i}^m N_i^m + \frac{\partial}{\partial E_i} (B_{i}^{mn} N_i^n) = Q_i^n + S_i^n - \sum_{m \neq n} \left\{ G_{i}^{mn} N_i^m + \frac{\partial}{\partial E_i} (B_{i}^{mn} N_i^m) \right\}, \]  

(C.5)

where

\[ Q_i^n = \frac{1}{h_h} \int_{-h_h}^{h_h} dz \cos \left( \frac{2n + 1 \pi z}{2 h_h} \right) Q_i, \]  

(C.6)

\[ S_i^n = \frac{1}{h_h} \int_{-h_h}^{h_h} dz \cos \left( \frac{2n + 1 \pi z}{2 h_h} \right) S_i, \]  

(C.7)

\[ G_{i}^{mn} = \frac{1}{h_h} \int_{-h_h}^{h_h} dz \cos \left( \frac{2n + 1 \pi z}{2 h_h} \right) \left( \frac{1}{\tau_i} + \frac{1}{\gamma \tau_{dec}} \right) \cos \left( \frac{2m + 1 \pi z}{2 h_h} \right), \]  

(C.8)

and

\[ B_{i}^{mn} = \frac{1}{h_h} \int_{-h_h}^{h_h} dz \cos \left( \frac{2n + 1 \pi z}{2 h_h} \right) \left( \frac{dE}{dt} \right) \cos \left( \frac{2m + 1 \pi z}{2 h_h} \right). \]  

(C.9)

By modeling the primary source distribution as

\[ Q_i(z) = Q_0 e^{-z/h_s}, \]  

(C.10)

\[ Q_i^n \] is calculated from eq. (C.6) [43]:

\[ Q_i^n = Q_0 g_1(h_h, h_s, n), \]  

(C.11)

\[ g_1(h_h, h_s, n) \equiv \frac{2h_s}{h_h} \left\{ 1 + \left( \frac{2n + 1 \pi h_s}{2 h_h} \right)^2 \right\}^{-1}. \]  

(C.12)

All the calculations in the text have been performed with

\[ h_s = 100 \text{ pc}. \]  

(C.13)

The secondary source term \( S_i \) is expressed as

\[ S_i(E_i) = \frac{4\pi \rho}{m_p} \sum_j CR \int dE_j \left[ I_j(E_j) \xi_{j-i} \frac{d\sigma_{j-p-i}}{dE_i} \right], \]  

(C.14)

(see also eq. (2.13), etc.)

where \( \rho = \rho(z) \) is the ISM density, whose spatial distribution is modeled as

\[ \rho(z) = \rho_0 e^{-z/h_g}. \]  

(C.15)

All the calculations have been performed with

\[ h_g = 100 \text{ pc}, \]  

(C.16)

\[ \rho_0 = 1.1 \text{ (atmos cm}^{-3} \text{)} \times m_{ISM}, \]  

(C.17)
where \( m_{\text{ISM}} \) is the mean mass of the ISM weighted by the number density. In eq. (C.14), the flux of parent species \( I_j(E_j) \) is assumed to be unchanged spatially. This approximation is valid because the spatial gradient of \( I_j \) is much smaller than that of \( \rho \). Then, from eq. (C.14), (C.15), and (C.7), we obtain

\[
S_i^n = \left\{ \frac{4\pi \rho_0}{m_p} \sum_j^{CR} \int dE_j \left[ I_j(E_j) \xi_{j-i} \frac{d\sigma_{jp-i}}{dE_i} \right] \right\} g_1(h, h_g, n) \quad \text{(C.18)}
\]

The interaction term is also dependent on \( z \) coordinate because it is written as

\[
\frac{1}{\tau_i} = \rho \beta c \frac{\xi_{i} \sigma_{ip}}{m_p},
\]

(see also eq. (2.16) etc)

where \( \rho \) is given by eq. (C.15). On the other hand, the intrinsic decay term \( 1/(\gamma \tau_{\text{dec}}) \) is not dependent on spatial coordinate. The coefficient \( G_{im} \) is then obtained as,

\[
G_{im} = \left( \rho_0 \beta c \frac{\xi_{i} \sigma_{ip}}{m_p} \right) g_2(h, h_g, n, m) + \delta_{nm} \frac{1}{\gamma \tau_{\text{dec}}},
\]

\[
g_2(h, h_g, n, m) = \frac{h_g}{h_h} \left[ \left\{ 1 + \left( (n + m + 1) \frac{\pi h_g}{h_h} \right)^2 \right\}^{-1} + \left\{ 1 + \left( (n - m) \frac{\pi h_g}{h_h} \right)^2 \right\}^{-1} \right].
\]

The energy loss term is expressed as

\[
\frac{dE}{dt} = \rho \beta c \left\langle \frac{dE}{dx} \right\rangle_{\text{ion}} + \left\langle \frac{dE}{dt} \right\rangle^e.
\]

The first term of the right-hand side corresponds to the ionization energy loss, which is dependent on \( z \) coordinate, while the second term represents the energy loss by synchrotron radiation and inverse Compton scattering (only for electrons/positrons, see eq. (5.1), Chapter 5), the rate of which is assumed to be independent of the spatial coordinate within the confinement volume. The coefficient \( B_{im} \) is then calculated as

\[
B_{im} = \rho_0 \beta c \left\langle \frac{dE}{dx} \right\rangle_{\text{ion}} g_2(h, h_g, n, m) + \delta_{nm} \left\langle \frac{dE}{dt} \right\rangle^e.
\]

By inserting thus obtained coefficients into eq. (C.5), we obtain

\[
\begin{align*}
\frac{D}{\rho \beta c} \left[ \frac{2n + 1 + \pi}{2h_h} \right]^2 I_i^n + g_2(h, h_g, n, n) &\frac{\xi_{i} \sigma_{ip} I_i^n}{m_p} + \frac{1}{\rho_0 \beta c \gamma \tau_{\text{dec}}} I_i^n \\
+ g_2(h, h_g, n, n) \frac{\partial}{\partial E_i} &\left( \left\langle \frac{dE}{dx} \right\rangle_{\text{ion}} I_i^n \right) + \frac{\partial}{\partial E_i} \left( \frac{1}{\rho_0 \beta c} \left\langle \frac{dE}{dt} \right\rangle^e I_i^n \right) \\
&= g_1(h, h_s, n) \frac{Q_{0i}}{4\pi \rho_0} + g_1(h, h_g, n) \frac{1}{m_p} \sum_j^{CR} \int dE_j \left[ I_j(E_j) \xi_{j-i} \frac{d\sigma_{jp-i}}{dE_i} \right] \\
&- \sum_{m \neq n} g_2(h, h_g, n, m) \left\{ \frac{\xi_{i} \sigma_{ip} I_i^n}{m_p} + \frac{\partial}{\partial E_i} \left( \left\langle \frac{dE}{dx} \right\rangle_{\text{ion}} I_i^n \right) \right\}
\end{align*}
\]

(24)
where
\[ I_i^n = \frac{\beta C}{4\pi} N_i^n, \quad (C.25) \]

(see also eq. (2.2))

Equation (C.24) could be solved if the last term of the right-hand side (\( \sum_{m \neq n} \) term) did not exist (see eq. (2.18)). Then we employ an iteration method as follows. First, \( I_i^0 \) is calculated using the solution (2.18) by neglecting the \( \sum_{m \neq n} \) term. Next, \( I_i^1 \) is also calculated using eq. (2.18). At this time, \( I_i^0 \) in the \( \sum_{m \neq n} \) term is inserted by the value already obtained and \( I_i^m (m \geq 2) \) terms are neglected. In this way, the “first-step values” of \( I_i^n \) are obtained up to large enough \( n \), say, \( n = 50 \). After that, \( I_i^0 \) is again calculated (the “second-step value”) using the first-step values of \( I_i^m (m \geq 1) \). The second-step value of \( I_i^1 \) is then calculated using the second-step value of \( I_i^0 \) and the first-step values of \( I_i^m (m \geq 2) \), and so on. With this method, solutions converge soon because the higher-order terms \( I_i^m (m \geq 1) \) are very small compared with the leading term \( I_i^0 (I_i^1 \text{ is } \sim 1/9 \text{ of } I_i^0) \) [43].

The diffusion coefficient \( D \) is related to the escape length as [10]
\[ \lambda_{esc} = \frac{\rho_0 \hbar h g \beta C}{D}, \quad (C.26) \]
then the escape term (the first term in the left-hand side of eq. (C.24)) is written using \( \lambda_{esc} \). Finally, we can obtain the equilibrium flux
\[ I_i(E_i, z) = \sum_{n=0}^{\infty} I_i^n(E_i) \cos \left( \frac{2n + 1}{2} \pi z \right). \quad (C.27) \]
At the galactic plane \((z = 0)\), the flux is simply
\[ I_i(E_i, 0) = \sum_{n=0}^{\infty} I_i^n(E_i), \quad (C.28) \]
which has been used in the text as a local interstellar flux near the Solar system.

The transport equation of the DR model with 1-D halo is
\[ \frac{\partial N_i}{\partial t} - D \frac{\partial^2 N_i}{\partial z^2} + \left( \frac{1}{\tau_i} + \frac{1}{\gamma \tau_{dec}} \right) N_i + \frac{\partial}{\partial E_i} \left( \frac{dE}{dt} N_i \right) \]
\[ + \frac{\partial}{\partial E_i} \left( \Gamma_{col} \langle \Delta E \rangle_{col} N_i \right) - \frac{\partial^2}{\partial E_i^2} \left( \frac{1}{2} \Gamma_{col} \langle (\Delta E)^2 \rangle_{col} N_i \right) = Q_i + S_i. \quad (C.29) \]

The above equation is the same as eq. (4.15) except for the last two terms of the left-hand side, which are added in order to express the Fermi acceleration effect. Because the Fermi acceleration is assumed to occur throughout the Galaxy, these two terms are independent of the spatial coordinate. Then we can treat them in
the same way as the $\langle dE/dt \rangle^e$ term, and the equation for each Fourier coefficient is obtained to be

$$\frac{dI_n^i}{dx} + \frac{D}{\rho_0 \beta c} \left( \frac{2n + 1}{2} \frac{\pi}{h_k} \right)^2 I_n^i + g_2(h_h, h_g, n, n_i) \frac{\xi_i \sigma_{ip}}{m_p} I_n^i + \frac{1}{\rho_0 \beta c \gamma \tau_{dec}} I_n^i$$

$$+ g_2(h_h, h_g, n, n) \frac{\partial}{\partial E_i} \left( \langle \frac{dE}{dx} \rangle_{\text{ion}} I_i^n \right) + \frac{\partial}{\partial E_i} \left( \frac{1}{\rho_0 \beta c} \langle \frac{dE}{dt} \rangle^e I_i^n \right)$$

$$+ \frac{\partial}{\partial E_i} \left( \frac{1}{\rho_0 \beta c} \Gamma_{\text{col}} \langle \Delta E \rangle_{\text{col}} I_i^n \right) - \frac{\partial^2}{\partial E_i^2} \left( \frac{1}{\rho_0 \beta c} \frac{1}{2} \Gamma_{\text{col}} \langle (\Delta E)^2 \rangle_{\text{col}} I_i^n \right)$$

$$= g_1(h_h, h_s, n) \frac{Q_{0i}}{4\pi \rho_0} + g_1(h_h, h_g, n) \frac{1}{m_p} \sum_j CR \int dE_j \left[ I_j(E_j) \xi_i \sigma_{ip} \frac{d\sigma_{ip}}{dE_i} \right]$$

$$- \sum_{m \neq n} g_2(h_h, h_g, n, m) \left\{ \frac{\xi_i \sigma_{ip} m_p}{I_i^m} \frac{\partial}{\partial E_i} \left( \langle \frac{dE}{dx} \rangle_{\text{ion}} I_i^n \right) \right\}.$$  \hspace{1cm} (C.30)

The equilibrium solution ($dI_n^i / dx = 0$) of the above equation is obtained numerically as follows. Initially, the solution of eq. (C.24) is inserted into $I_n^i$ of eq. (C.30), and $dI_n^i / dx$ is calculated in order to obtain the new value of $I_n^i$ by adding the current value to $(dI_n^i / dx) \Delta x$, where $\Delta x$ is the small step of traversed matter. This procedure is repeated until $dI_n^i / dx$ become zero (very small) [3]. The solution obtained using this method is not dependent on the step $\Delta x$, and the cumulative effect of the small errors does not exist [3]. Although not shown here, we have also obtained the solution of the DR model using the method shown by Heinbach and Simon [24]. We have confirmed that the two solutions are in good agreement within $\pm 5\%$. 

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References


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