Low energy neutrino experiments sensitivity to physics beyond the Standard Model

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Outline

- Motivations
- Non-standard ν-e and ν-q interactions
- Non-standard contributions to ν-N coherent scattering
- Sensitivity to specific NSI scenarios:
  Z', leptoquark and R-parity breaking SUSY
- Weak mixing angle and neutrino charge radius
- Summary

References:

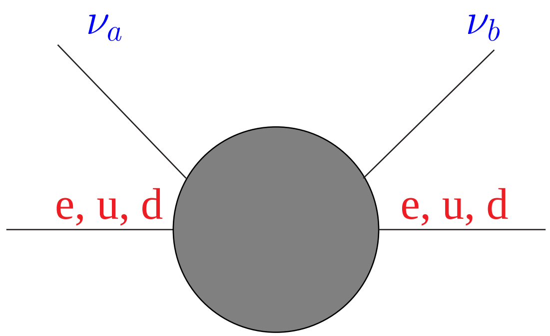
J. Barranco, O. Miranda and TR, hep-ph/0508299, hep-ph/0702175, arXiv:0707.4319

Non Standard Interactions (NSI)

Most extensions of the SM, in particular neutrino mass theories, predict neutral current non-standard interactions (NSI) of neutrinos which can be either flavor preserving (NU – non-universal) or flavor-changing (FC).

NSI effective Lagragian form:

$$\mathcal{L}_{eff}^{NSI} = -\sum_{\alpha\beta f P} \varepsilon_{\alpha\beta}^{fP} 2\sqrt{2} G_F (\bar{\nu}_\alpha \gamma_\rho L \nu_\beta)(\bar{f}\gamma^\rho P f)$$



Here $\alpha, \beta = e, \mu, \tau$; $f = e, u, d$; $P = L, R$; $L = (1-\gamma_5)/2$; $R = (1+\gamma_5)/2$

Non Standard Interactions (NSI)

Non-standard neutral current neutrino interactions may arise:

- from a non trivial non-unitary lepton mixing matrix
  Schechter & Valle'80
- in models where neutrino masses are "calculable" from radiative corrections
  Zee'80, Babu'88
- in SUSY models with broken R-parity
  see review by Hirsch & Valle [hep-ph/0405015] and refs therein
- in unified SUSY models as a renormalization effect
  Hall, Kostelecky & Raby'86
- . . . some other models, like left-right models, etc . . .

Predictions:

In most models NSI contributions are expected to be small, e.g. being supressed by the smallness of neutrino masses, however in some models NSI is not strongly restricted

Current bounds on NSI couplings

Bounds on NSI couplings come from

- ν-scattering experiments: LSND, CHARM, NuTeV, MUNU, MINOS

  Barger et al'91, Davidson et al'03, Barranco et al'05

  Friedland et al'06

- $e^- e^+ \to \nu\bar{\nu}\gamma$ measured at LEP

  Berezhiani & Rossi'02

- analysis of atmospheric neutrino data

  Fornengo et al'02, Friedland et al'04'05

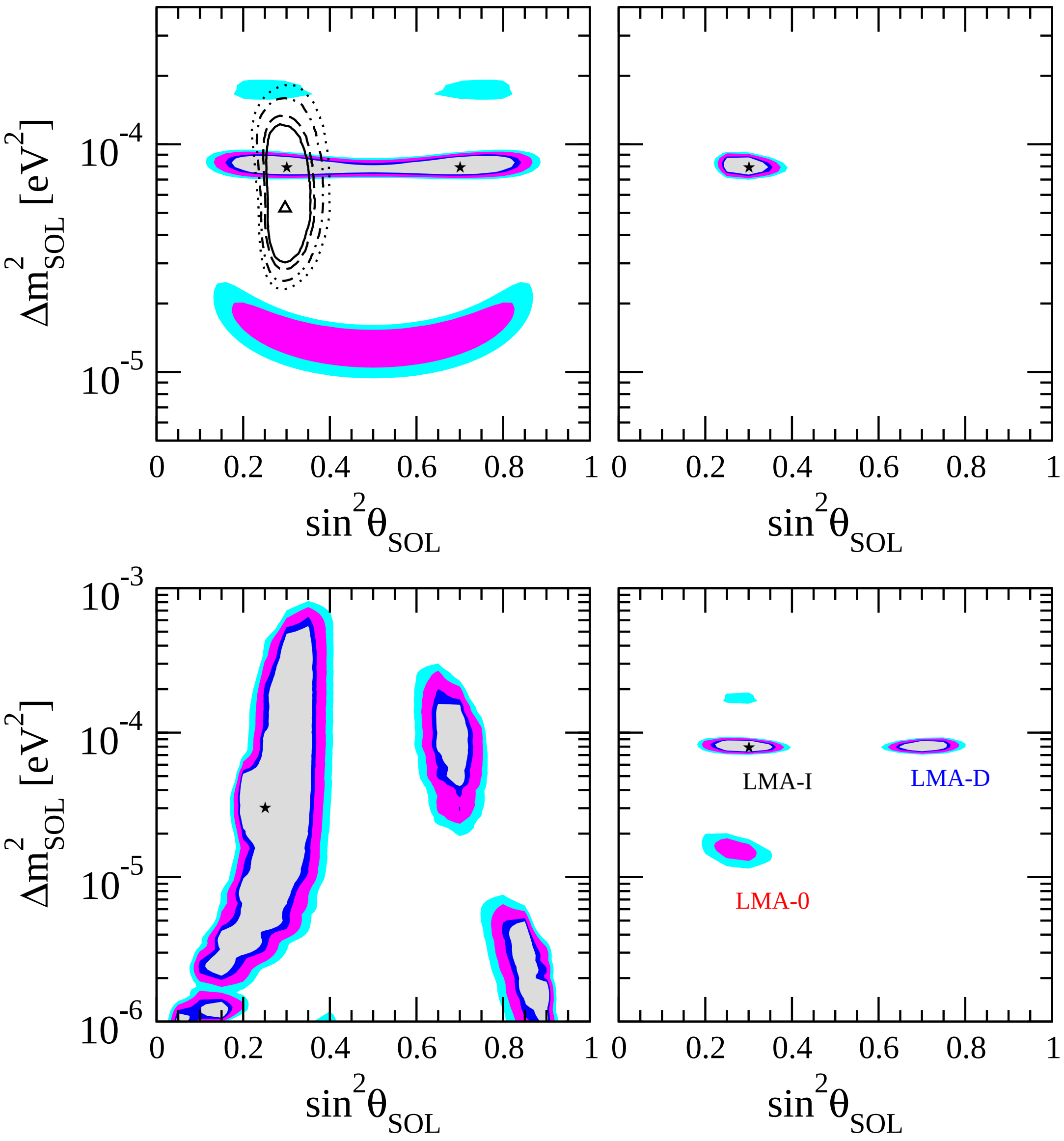
- lepton flavor violating interactions, appeared at loop level from NSI, like μ capture by nuclei

  Davidson et al'03

- Invisible Z-boson decay width including loop corrections due to NSI

  Davidson et al'03

Solar + KamLAND without and with NSI



Miranda, Tortola, Valle, hep-ph/0406280

Applications (not complete list!)

- ν oscillations in matter

  . . . , Guzzo et al'91, Fornengo et al'02, Friedland et al'04'05, Miranda et al'04, Kopp et al'07

- ν scattering experiments

  . . . , Barger et al'91, Davidson et al'03, Barranco et al'05, Kopp et al'07

- supernovae explosion

  Freedman et al'77, Fuller et al'87'88, Amanik et al'04'06, Esteban-Pretel et al'07

- LEP (ILC)

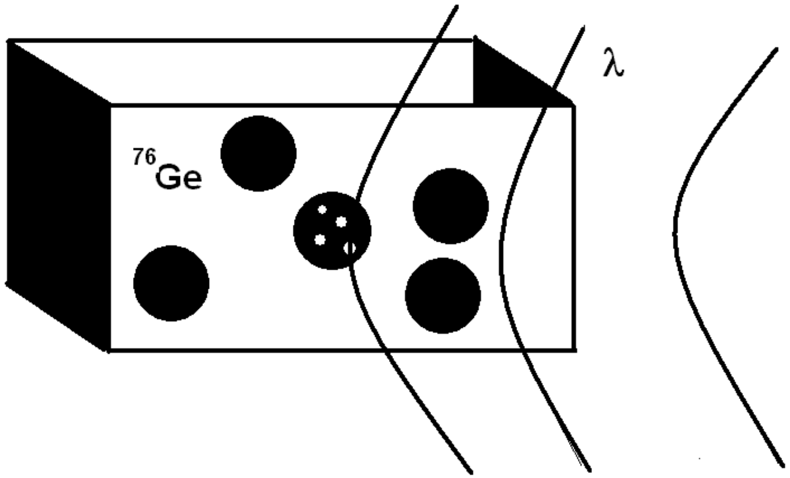
  Berezhiani & Rossi'02, Davidson et al'03

- Early Universe Mangano et al'06

- . . .

Coherent neutrino scattering off nuclei (Henry Wong talk yesterday!)

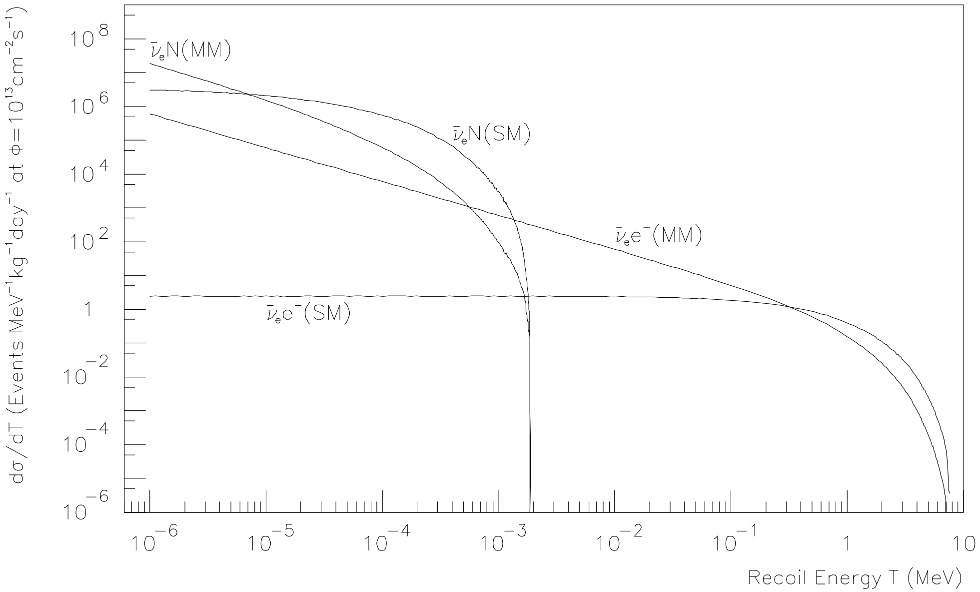
- Good statistics due to quadratic coherent enhancement
- Sensitivity to ν-quark couplings
- Coherent scattering if the momentum transfer, Q, is small, $QR < 1$ (R is radius of nucleus): $\Longrightarrow$ ν-s doesn't "see" structure of nucleus!
- For most of nuclei: $1/R \sim 25 - 150$ MeV
- Well satisfied for most neutrino sources like supernovae, solar, reactor and artificial neutrino sources
- Planned experiments to measure coherent ν-N scattering: NOSTOS, TEXONO ...and other proposals
- Experimentally difficult: very low energy threshold



Proposed experiments to measure coherent ν-N scattering

- TEXONO: 1kg of germanium, reactor neutrinos

  hep-ex/0511001, H.Wong talk yesterday
- NOSTOS: spherical TPC detector, 10 ton of Xenon — astro-ph/0511470
- Stopped-π ν beam and kg-to-ton mass detector — hep-ex/0511042
- beta-beams — Bueno et al, PRD'06
- more ideas in the past,
  - superconducting detector (Drukier & Stodolsky'84)
  - acoustic (Krauss'91)
  - cryogenic (Oberauer'02)



ν-N coherent scattering

$$\frac{d\sigma}{dT} = \frac{G_F^2 M}{2\pi}\left\{(G_V+G_A)^2+(G_V-G_A)^2\left(1-\frac{T}{E_\nu}\right)^2-(G_V^2-G_A^2)\,\frac{MT}{E_\nu^2}\right\}$$

$$G_V = \left[\left(g_V^p+2\varepsilon_{ee}^{uV}+\varepsilon_{ee}^{dV}\right)Z+\left(g_V^n+\varepsilon_{ee}^{uV}+2\varepsilon_{ee}^{dV}\right)N\right]F_{nucl}^V(Q^2)$$

$$G_A = \left[\left(g_A^p+2\varepsilon_{ee}^{uA}+\varepsilon_{ee}^{dA}\right)(Z_+-Z_-)+\left(g_A^n+\varepsilon_{ee}^{uA}+2\varepsilon_{ee}^{dA}\right)(N_+-N_-)\right]F_{nucl}^A(Q^2)$$

$$\begin{aligned}\frac{d\sigma}{dT}(E_\nu,T) &= \frac{G_F^2 M}{\pi}\left(1-\frac{MT}{2E_\nu^2}\right)\times\\ &\times \left\{\left[Z(g_V^p+2\varepsilon_{ee}^{uV}+\varepsilon_{ee}^{dV})+N(g_V^n+\varepsilon_{ee}^{uV}+2\varepsilon_{ee}^{dV})\right]^2+\right.\\ &+ \left.\sum_{\alpha=\mu,\tau}\left[Z(2\varepsilon_{\alpha e}^{uV}+\varepsilon_{\alpha e}^{dV})+N(\varepsilon_{\alpha e}^{uV}+2\varepsilon_{\alpha e}^{dV})\right]^2\right\}\end{aligned}$$

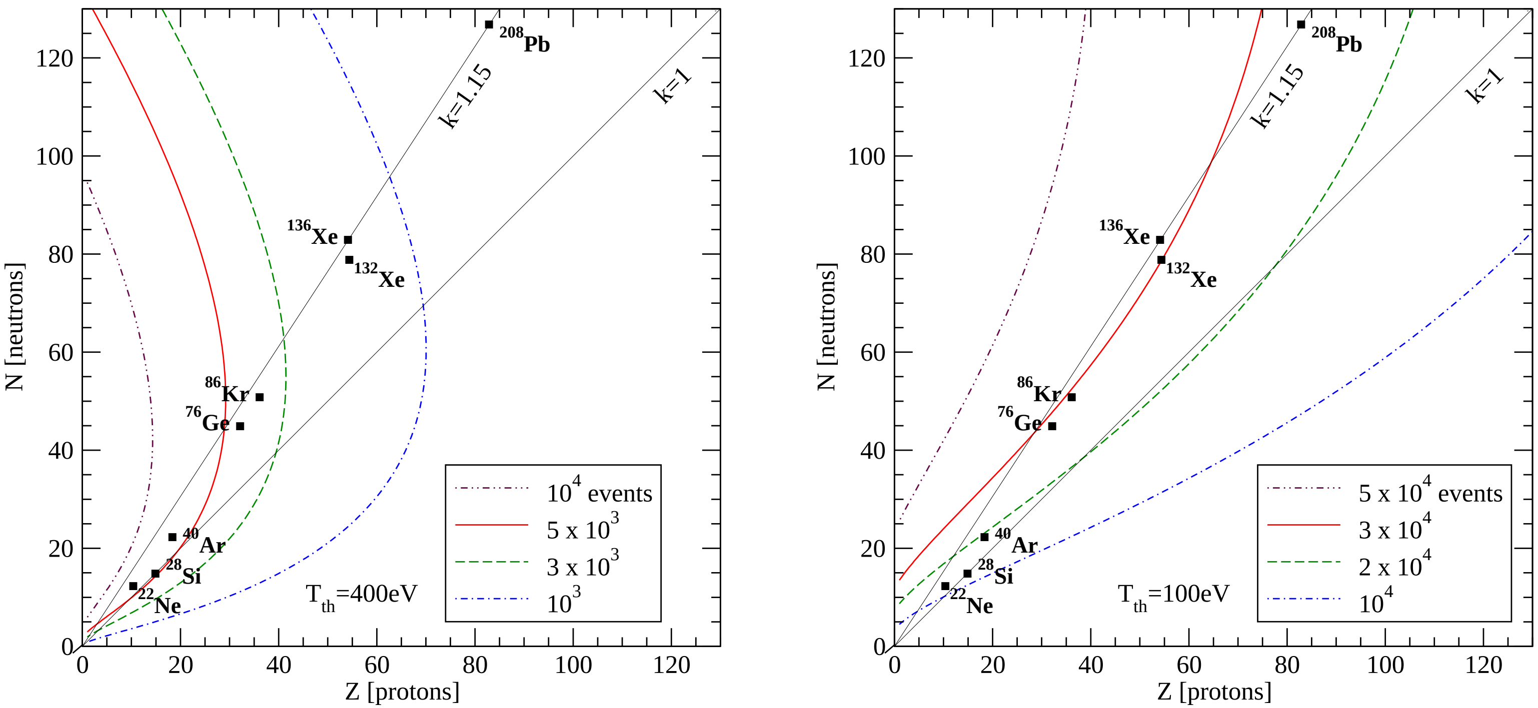
- Axial couplings contribution is zero or can be neglected
- Coherent enhancement of cross section
- Degeneracy in determination of NSI parameters

Resolving degeneracy

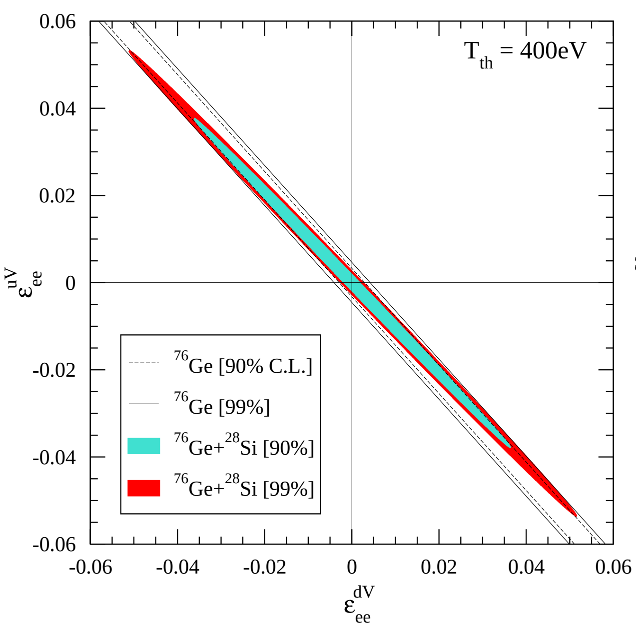
$$\left[Z(g_V^p + 2\varepsilon_{ee}^{uV} + \varepsilon_{ee}^{dV}) + N(g_V^n + \varepsilon_{ee}^{uV} + 2\varepsilon_{ee}^{dV})\right]^2 = \left[Zg_V^p + Ng_V^n\right]^2$$

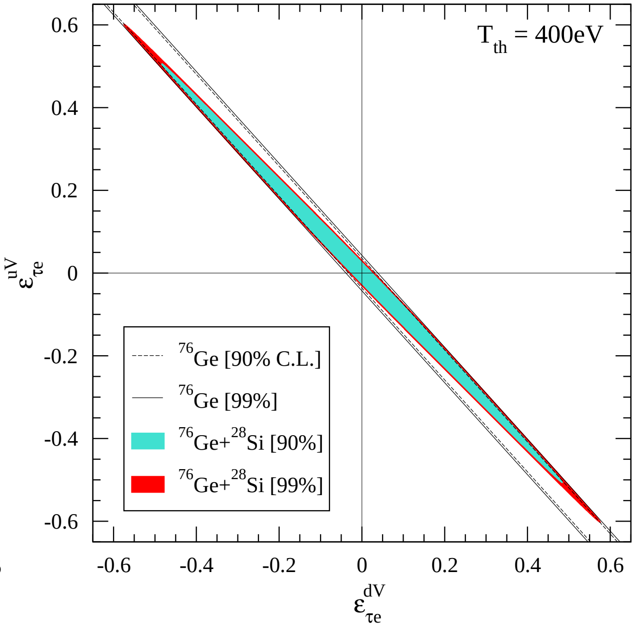
$$\varepsilon_{ee}^{uV}(A+Z) + \varepsilon_{ee}^{dV}(A+N) = \text{const}\,.$$

Solution: take two targets with **maximally different** $k = (A+N)/(A+Z)$

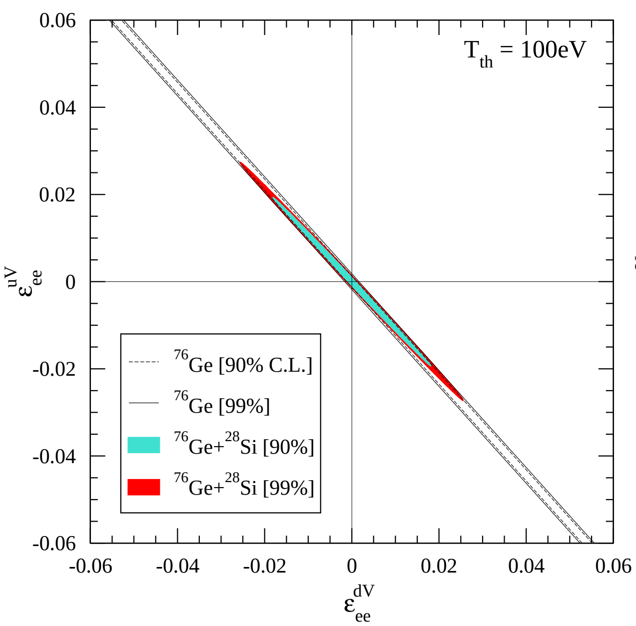


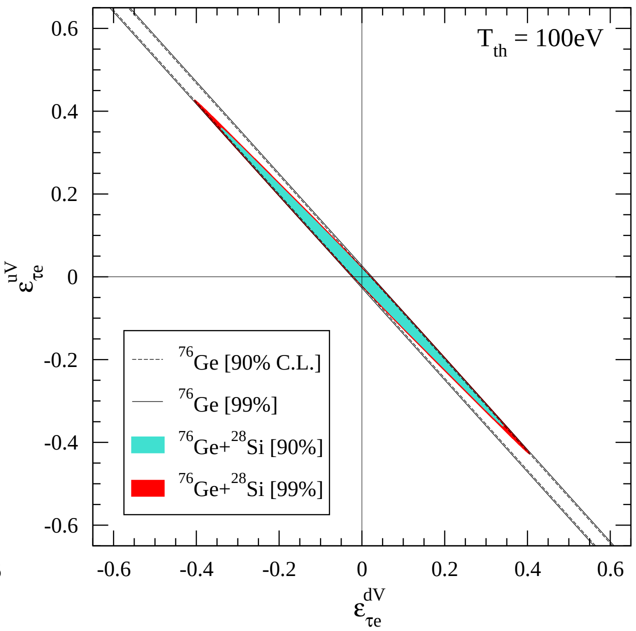
Estimated bounds on NSI from TEXONO-like experiment (Ge+Si)





^{76}Ge+^{28}Si $T_{th}=400eV$
$\|\epsilon_{ee}^{dV}\| < 0.036$
$\|\epsilon_{ee}^{uV}\| < 0.038$
$\|\epsilon_{\tau e}^{dV}\| < 0.48$
$\|\epsilon_{\tau e}^{uV}\| < 0.50$





^{76}Ge+^{28}Si $T_{th}=100eV$
$\|\epsilon_{ee}^{dV}\| < 0.018$
$\|\epsilon_{ee}^{uV}\| < 0.019$
$\|\epsilon_{\tau e}^{dV}\| < 0.34$
$\|\epsilon_{\tau e}^{uV}\| < 0.37$

Present bounds and future sensitivity to NSI

One parameter analysis to compare coherent scattering sensitivity with present bounds and νFactory sensitivity (from Davidson et al'03), see also Kropp et al, hep-ph/0702269

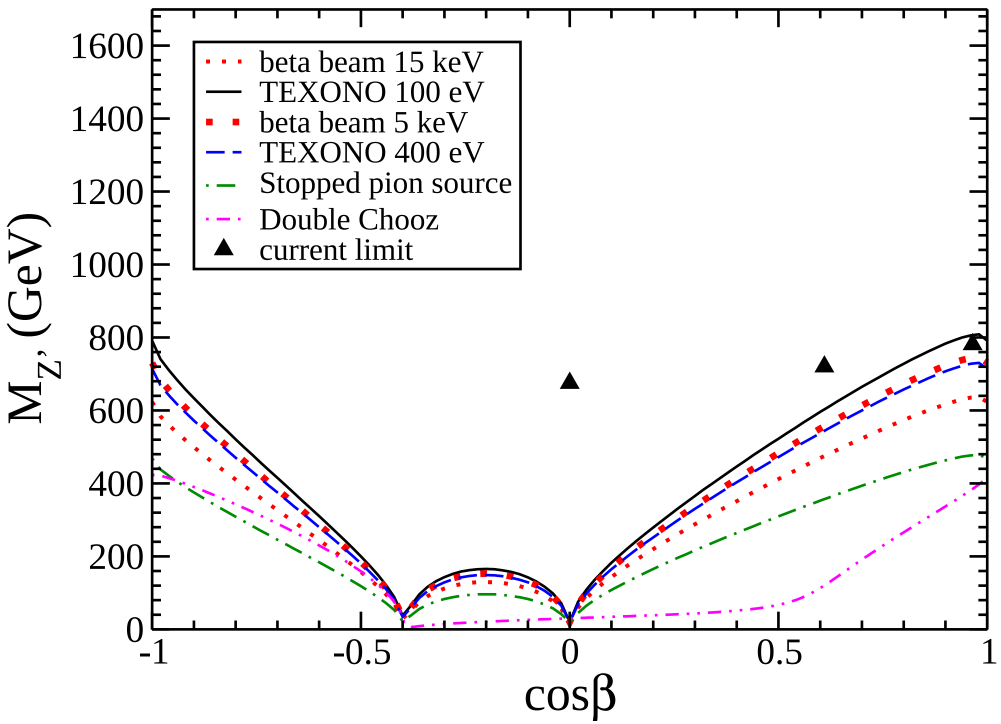
	Present Limits	ν Factory	^{76}Ge $_{T_{th}=400eV}$ (^{76}Ge $_{T_{th}=100eV}$)	^{76}Ge+^{28}Si $_{T_{th}=400eV}$ (^{76}Ge+^{28}Si $_{T_{th}=100eV}$)
ϵ_{ee}^{dV}	$-0.5 < \epsilon_{ee}^{dV} < 1.2$	$\lvert\epsilon_{ee}^{dV}\rvert < 0.002$	$\lvert\epsilon_{ee}^{dV}\rvert < 0.003$ ($\lvert\epsilon_{ee}^{dV}\rvert < 0.001$)	$\lvert\epsilon_{ee}^{dV}\rvert < 0.002$ ($\lvert\epsilon_{ee}^{dV}\rvert < 0.001$)
$\epsilon_{\tau e}^{dV}$	$\lvert\epsilon_{\tau e}^{dV}\rvert < 0.78$	$\lvert\epsilon_{\tau e}^{dV}\rvert < 0.06$	$\lvert\epsilon_{\tau e}^{dV}\rvert < 0.032$ ($\lvert\epsilon_{\tau e}^{dV}\rvert < 0.020$)	$\lvert\epsilon_{\tau e}^{dV}\rvert < 0.024$ ($\lvert\epsilon_{\tau e}^{dV}\rvert < 0.017$)
ϵ_{ee}^{uV}	$-1.0 < \epsilon_{ee}^{uV} < 0.61$	$\lvert\epsilon_{ee}^{uV}\rvert < 0.002$	$\lvert\epsilon_{ee}^{uV}\rvert < 0.003$ ($\lvert\epsilon_{ee}^{uV}\rvert < 0.001$)	$\lvert\epsilon_{ee}^{uV}\rvert < 0.002$ ($\lvert\epsilon_{ee}^{uV}\rvert < 0.001$)
$\epsilon_{\tau e}^{uV}$	$\lvert\epsilon_{\tau e}^{uV}\rvert < 0.78$	$\lvert\epsilon_{\tau e}^{uV}\rvert < 0.06$	$\lvert\epsilon_{\tau e}^{uV}\rvert < 0.036$ ($\lvert\epsilon_{\tau e}^{uV}\rvert < 0.023$)	$\lvert\epsilon_{\tau e}^{uV}\rvert < 0.023$ ($\lvert\epsilon_{\tau e}^{uV}\rvert < 0.018$)

Specific NSI scenarios

Extra heavy neutral gauge boson Z'

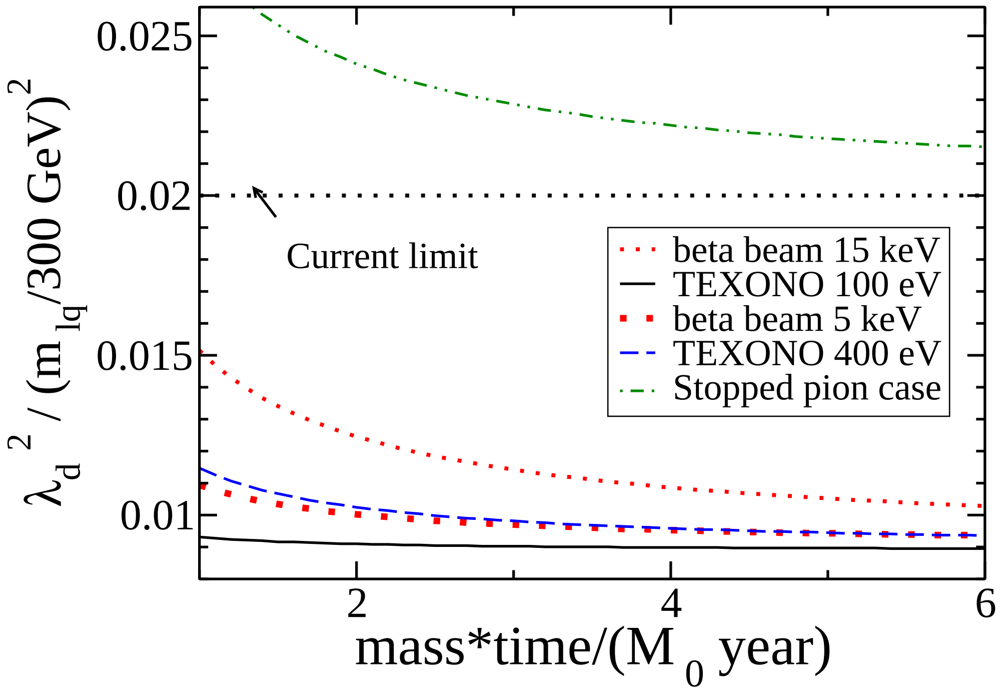
$$\varepsilon_{ee}^{uL} = \varepsilon_{ee}^{dL} = -\varepsilon_{ee}^{uR} = -4\gamma \sin^2\theta_W \rho_{\nu N}^{NC} \left(\frac{c_\beta}{\sqrt{24}} - \frac{s_\beta}{3}\sqrt{\frac{5}{8}}\right)\left(\frac{3c_\beta}{2\sqrt{24}} + \frac{s_\beta}{6}\sqrt{\frac{5}{8}}\right)$$

$$\varepsilon_{ee}^{dR} = -8\gamma \sin^2\theta_W \rho_{\nu N}^{NC} \left(\frac{3c_\beta}{2\sqrt{24}} + \frac{s_\beta}{6}\sqrt{\frac{5}{8}}\right)^2, \quad \gamma = (M_Z/M_{Z'})^2$$



Leptoquark

$$\varepsilon^{uV} = \frac{\lambda_u^2}{m_{lq}^2}\frac{\sqrt{2}}{4G_F}, \qquad \varepsilon^{dV} = \frac{\lambda_d^2}{m_{lq}^2}\frac{\sqrt{2}}{4G_F}$$

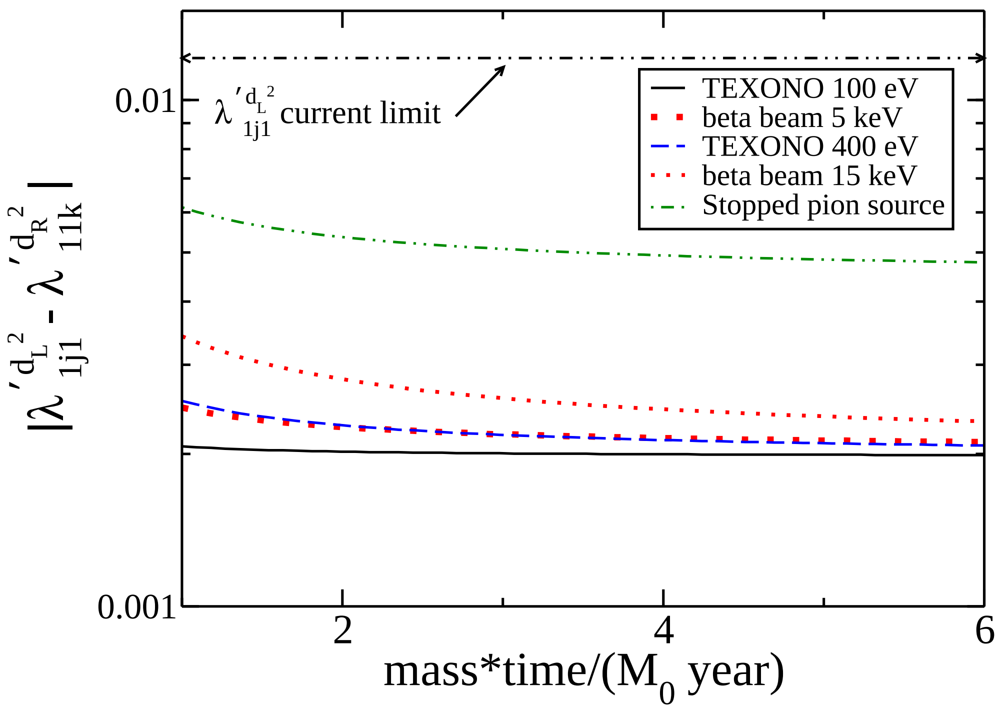


In some models leptoquark can be light See Dorsner et al'05

SUSY with broken R-parity

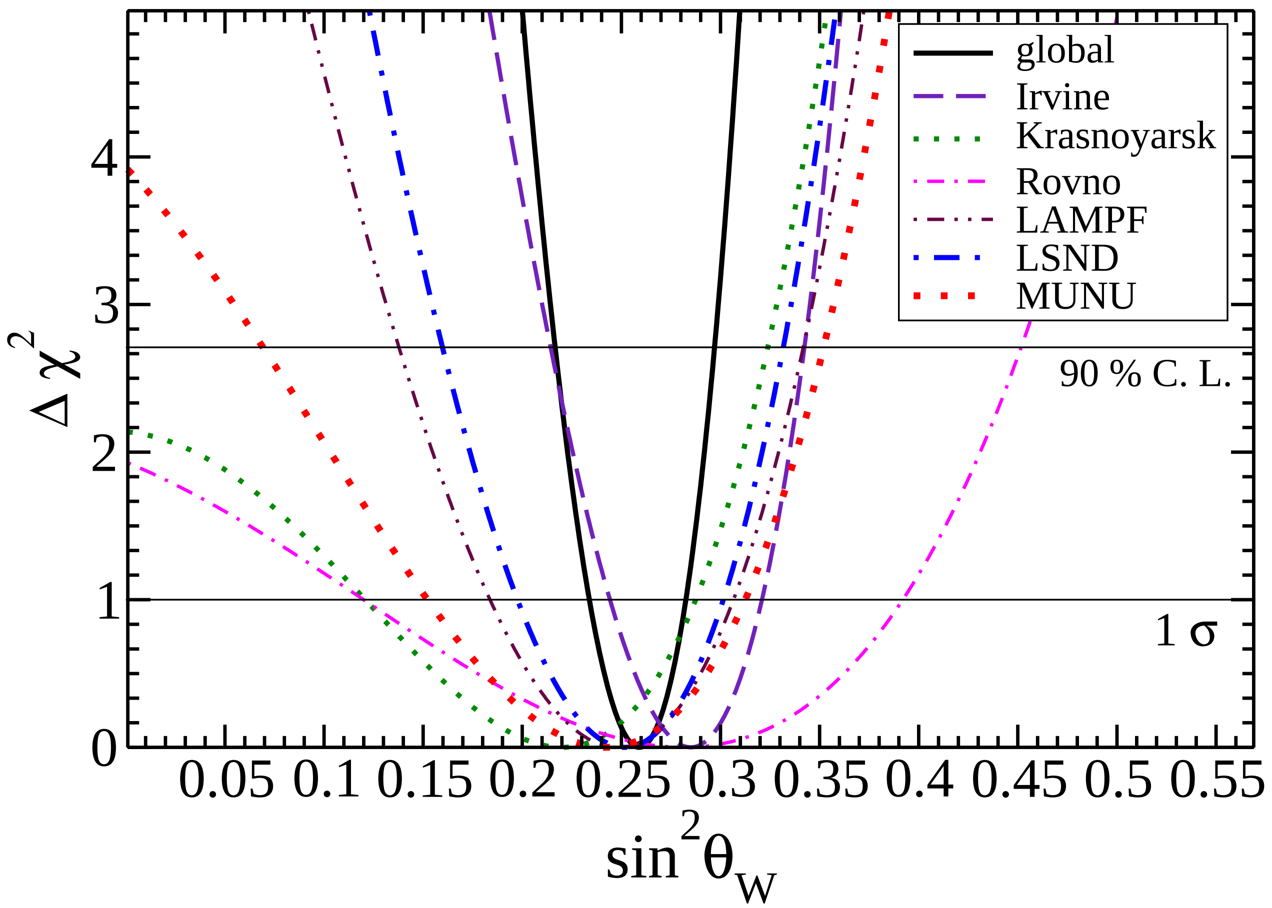
R-parity violating MSSM (imposing baryon number conservation) with a superpotential that contains the following L- violating terms:

$$\lambda_{ijk} L_L^i L_L^j \bar{E}_R^k, \quad \lambda'_{ijk} L_L^i Q_L^j \bar{D}_R^k$$



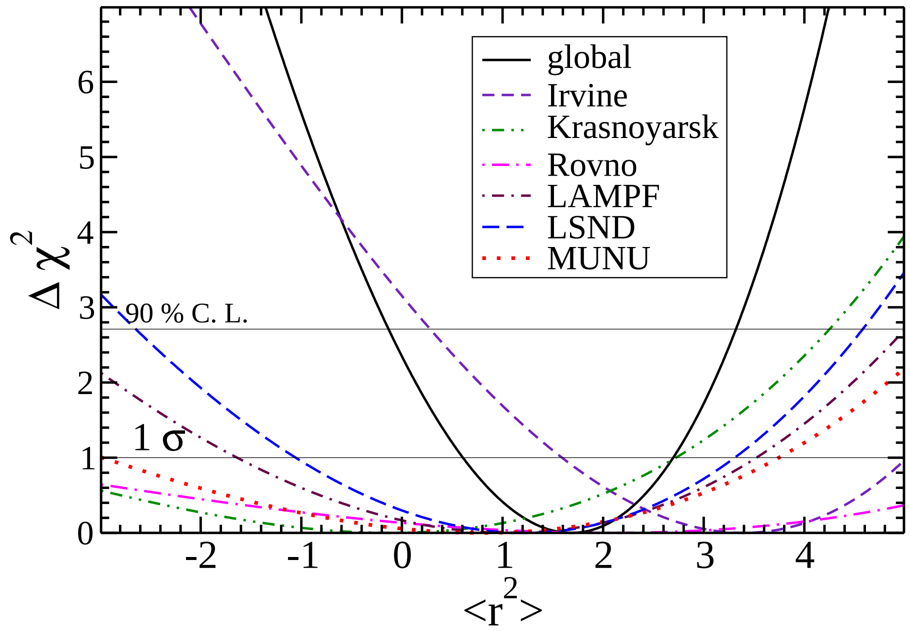
The $\nu_e e$ interaction and $\sin^2\theta_W$

The $\nu_e e$ interaction and $\sin^2\theta_W$



- Barranco, Miranda, TR, arXiv:0707.4319: $\sin^2\theta_W = 0.259 \pm 0.025$
- not competitive to NuTeV ($\sin^2\theta_W$(on-shell)=$0.2277 \pm 0.0013 \pm 0.0009$), Moller scattering and atomic parity violation (1%), but different channel: $\nu_e e$ scattering!

The $\nu_e e$ interaction and a ν_e effective charge radius



- The existence of a non-zero neutrino charge radius as a gauge-independent quantity has been the subject of a recent polemic: J. Bernabeu, J. Papavassiliou and J. Vidal, Phys. Rev. Lett. **89**, 101802 (2002); arXiv:hep-ph/0303202. K. Fujikawa and R. Shrock, Phys. Rev. D **69**, 013007 (2004); arXiv:hep-ph/0303188.
- If the neutrino charge radius exists it should have a value of $\approx 0.4 \times 10^{-32}$ cm^2 and it would modify the $\nu_e e$ by changing the value of $\sin^2\theta_W$ to $\sin^2\theta_W = \sin^2\overline{\theta_W} + \delta$ with the radiative correction $\delta = (\sqrt{2}\pi\alpha/3G_F)\langle r^2_{\nu_e}\rangle = 2.3796 \times 10^{30}$ cm^2 $\times \langle r^2_{\nu_e}\rangle$
our result: -0.13×10^{-32} cm^2 $< \langle r^2_{\nu_e}\rangle < 3.32 \times 10^{-32}$ cm^2 at 90% C.L.

ν_e-e scattering, $\sin^2\theta_W$ and $\langle r^2_{\nu_e}\rangle$

Experiment	Energy MeV	Events	$\sin^2\theta_W$	$\langle r^2_{\nu_e}\rangle >$ 10^{-32} cm^2 90% C.L.	$\langle r^2_{\nu_e}\rangle <$ 10^{-32} cm^2 90% C.L.
LAMPF	$7-60$	236	0.249 ± 0.063	-3.56	5.44
LSND	$10-50$	191	0.248 ± 0.051	-2.97	4.14
Irvine	$1.5-3.0$	381	0.29 ± 0.05	N/A	N/A
	$3.0-4.5$	77			
Krasnoyarsk	$3.15-5.175$	N/A	$0.22^{+0.7}_{-0.8}$	-7.3	7.3
Rovno	$0.6-2.0$	41	N/A	N/A	N/A
MUNU	$0.7-2.0$	68	N/A	N/A	N/A
Global			0.259±0.025	-0.13	3.32

Conclusions

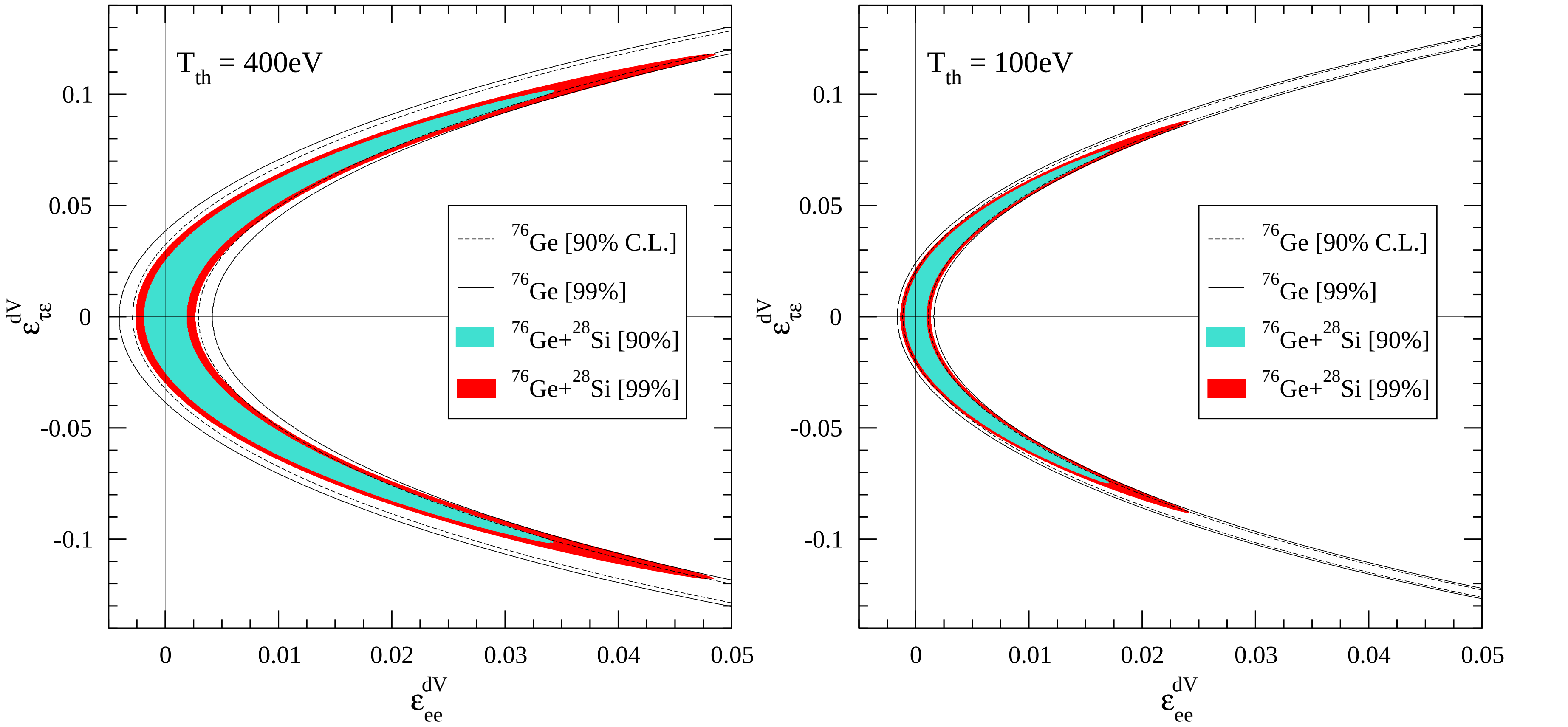
- Non-standard interactions in the neutrino sector are predicted in models beyond the SM and can be rather large
- NSI can play significant role in astrophysical environments
- Coherent neutrino scattering off nuclei gives a new sensitive probe to ν-quark vector couplings
- Combined analysis of all $\nu_e e$ scattering experiments gives new limit to ν_e effective charge radius

neutrino-nuclei scattering

	^{76}Ge+^{28}Si $_{T_{th}=400eV}$	^{76}Ge+^{28}Si $_{T_{th}=100eV}$
ϵ_{ee}^{dV}	$\lvert\epsilon_{ee}^{dV}\rvert < 0.036$	$\lvert\epsilon_{ee}^{dV}\rvert < 0.018$
ϵ_{ee}^{uV}	$\lvert\epsilon_{ee}^{uV}\rvert < 0.038$	$\lvert\epsilon_{ee}^{uV}\rvert < 0.019$
$\epsilon_{\tau e}^{dV}$	$\lvert\epsilon_{\tau e}^{dV}\rvert < 0.48$	$\lvert\epsilon_{\tau e}^{dV}\rvert < 0.34$
$\epsilon_{\tau e}^{uV}$	$\lvert\epsilon_{\tau e}^{uV}\rvert < 0.50$	$\lvert\epsilon_{\tau e}^{uV}\rvert < 0.37$
ϵ_{ee}^{dV}	$-0.002 < \epsilon_{ee}^{dV} < 0.034$	$-0.0009 < \epsilon_{ee}^{dV} < 0.016$
$\epsilon_{\tau e}^{dV}$	$\lvert\epsilon_{\tau e}^{dV}\rvert < 0.1$	$\lvert\epsilon_{\tau e}^{dV}\rvert < 0.074$

NSI with d-quark only

$$\varepsilon^{dV}_{\tau e} \quad \text{versus} \quad \varepsilon^{dV}_{ee}$$



Example: $\not{R}_p$ parity violating SUSY

Non-standard neutrino-electron and neutrino quark interactions:

$$\mathcal{L} = \lambda_{ijk} \tilde{e}_R^{k*} (\bar{\nu}_L^i)^c e_L^j + \lambda'_{ijk} \tilde{d}_L^j \bar{d}_R^k \nu_L^i + \dots$$

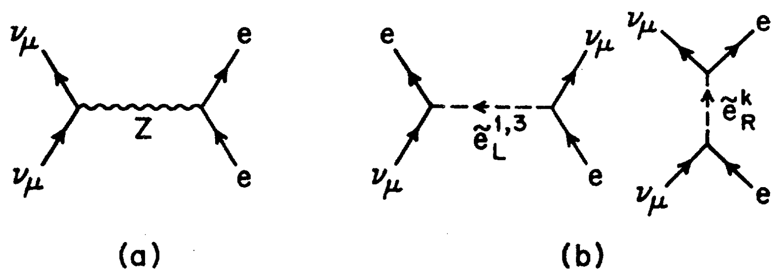
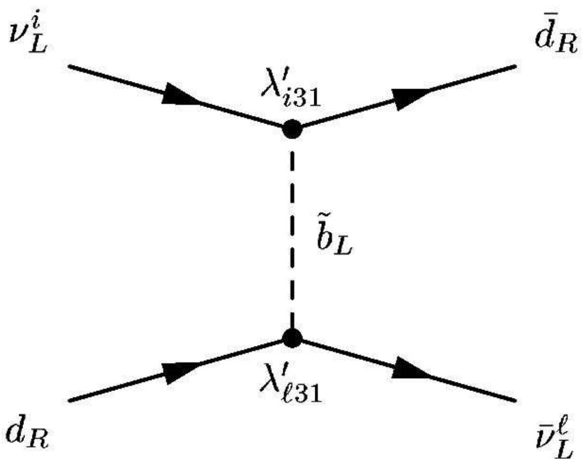
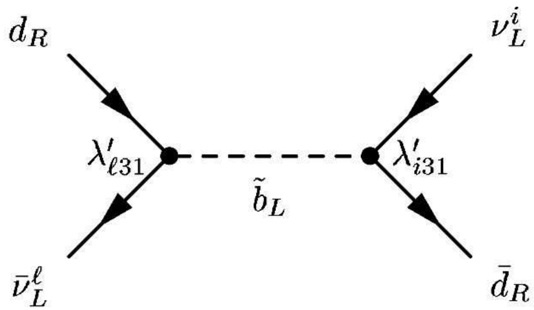


FIG. 2. Feynman diagrams for $\nu_\mu e$ scattering from (a) the standard model, and (b) the R-breaking interactions.

Barger, Giudice & Han'89





See e.g. Roulet'91, Amanik et al'05

Solar ν oscillations and NSI

$$H_{\rm NSI} = \sqrt{2} G_F N_f \begin{pmatrix} 0 & \varepsilon \\ \varepsilon & \varepsilon' \end{pmatrix} .$$

with,

$$\varepsilon = -\sin\theta_{23}\, \varepsilon^{fV}_{e\tau} \qquad \varepsilon' = \sin^2\theta_{23}\, \varepsilon^{fV}_{\tau\tau} - \varepsilon^{fV}_{ee}$$

and

$$\varepsilon^{fV}_{\tau\tau} = \varepsilon^{fL}_{\tau\tau} + \varepsilon^{fR}_{\tau\tau}$$

Note:

ν oscillations are sensitive mainly to vector couplings because matter is not polarized.

Current bounds on NU NSI ν-q couplings from Davidson et al'03

vertex	current limits	future limit
$(\bar{u}\gamma^\rho Pu)(\bar{\nu}_\tau\gamma_\rho L\nu_\tau)$	$\lvert\varepsilon^{uL}_{\tau\tau}\rvert < 1.4$ $\lvert\varepsilon^{uR}_{\tau\tau}\rvert < 3$ $(\Gamma_{inv})^{*)}$	$-0.3 < \varepsilon^{uL}_{\tau\tau} < 0.25$ $-0.25 < \varepsilon^{uR}_{\tau\tau} < 0.3$ KamLAND and SNO/SK
$(\bar{d}\gamma^\rho Ld)(\bar{\nu}_\tau\gamma_\rho L\nu_\tau)$	$\lvert\varepsilon^{dL}_{\tau\tau}\rvert < 1.1$ $\lvert\varepsilon^{dR}_{\tau\tau}\rvert < 6$ $(\Gamma_{inv})^{*)}$	$-0.25 < \varepsilon^{dL}_{\tau\tau} < 0.3$ $-0.3 < \varepsilon^{dR}_{\tau\tau} < 0.25$ KamLAND and SNO/SK
$(\bar{u}\gamma^\rho Pu)(\bar{\nu}_\mu\gamma_\rho L\nu_\mu)$	$\lvert\varepsilon^{uL}_{\mu\mu}\rvert < 0.003$ $-0.008 < \varepsilon^{uR}_{\mu\mu} < 0.003$ NuTeV	$\lvert\varepsilon^{uL}_{\mu\mu}\rvert < 0.001$ $\lvert\varepsilon^{uR}_{\mu\mu}\rvert < 0.002$ s^2_W in DIS at νFactory
$(\bar{d}\gamma^\rho Pd)(\bar{\nu}_\mu\gamma_\rho L\nu_\mu)$	$\lvert\varepsilon^{dL}_{\mu\mu}\rvert < 0.003$ $-0.008 < \varepsilon^{dR}_{\mu\mu} < 0.015$ NuTeV	$\lvert\varepsilon^{dL}_{\mu\mu}\rvert < 0.0009$ $\lvert\varepsilon^{dR}_{\mu\mu}\rvert < 0.005$ s^2_W in DIS at νFactory
$(\bar{u}\gamma^\rho Pu)(\bar{\nu}_e\gamma_\rho L\nu_e)$	$-1 < \varepsilon^{uL}_{ee} < 0.3$ $-0.4 < \varepsilon^{uR}_{ee} < 0.7$ CHARM	$\lvert\varepsilon^{uL}_{ee}\rvert < 0.001$ $\lvert\varepsilon^{uR}_{ee}\rvert < 0.002$ s^2_W in DIS at νFactory
$(\bar{d}\gamma^\rho Pd)(\bar{\nu}_e\gamma_\rho L\nu_e)$	$-0.3 < \varepsilon^{dL}_{ee} < 0.3$ $-0.6 < \varepsilon^{dR}_{ee} < 0.5$ CHARM	$\lvert\varepsilon^{dL}_{ee}\rvert < 0.0009$ $\lvert\varepsilon^{dR}_{ee}\rvert < 0.005$ s^2_W in DIS at νFactory

Current bounds on FC NSI ν-q couplings from Davidson et al'03

vertex	current limits	future limit
$(\bar{u}\gamma^\rho Pu)(\bar{\nu}_\tau\gamma_\rho L\nu_\mu)$	$\lvert\varepsilon^{uP}_{\tau\mu}\rvert < 0.05$ NuTeV	$\lvert\varepsilon^{uP}_{\tau\mu}\rvert < 0.03$ s^2_W in DIS at νFactory
$(\bar{d}\gamma^\rho Pd)(\bar{\nu}_\tau\gamma_\rho L\nu_\mu)$	$\lvert\varepsilon^{dP}_{\tau\mu}\rvert < 0.05$ NuTeV	$\lvert\varepsilon^{dP}_{\tau\mu}\rvert < 0.03$ s^2_W in DIS at νFactory
$(\bar{u}\gamma^\rho Pu)(\bar{\nu}_\mu\gamma_\rho L\nu_e)$	$\lvert\varepsilon^{uP}_{\mu e}\rvert < 7.7 \times 10^{-4}$ $(\mathrm{Ti}\mu \to \mathrm{Ti}e)^{*)}$	
$(\bar{d}\gamma^\rho Pd)(\bar{\nu}_\mu\gamma_\rho L\nu_e)$	$\lvert\varepsilon^{dP}_{\mu e}\rvert < 7.7 \times 10^{-4}$ $(\mathrm{Ti}\mu \to \mathrm{Ti}e)^{*)}$	
$(\bar{u}\gamma^\rho Pu)(\bar{\nu}_\tau\gamma_\rho L\nu_e)$	$\lvert\varepsilon^{uP}_{\tau e}\rvert < 0.5$ CHARM	$\lvert\varepsilon^{uP}_{\tau e}\rvert < 0.03$ s^2_W in DIS at νFactory
$(\bar{d}\gamma^\rho Pd)(\bar{\nu}_\tau\gamma_\rho L\nu_e)$	$\lvert\varepsilon^{dP}_{\tau e}\rvert < 0.5$ CHARM	$\lvert\varepsilon^{dP}_{\tau e}\rvert < 0.03$ s^2_W in DIS at νFactory

- All these bounds are derived taking one parameter at a time!
- NSI couplings with ν_μ are already strongly restricted
- See Davidson et al'03 [hep-ph/0302093] for NSI neutrino-electron couplings bounds

Neutrino-nuclei interaction

$$\mathcal{L}^{NSI}_{\nu Hadron} = -\frac{G_F}{\sqrt{2}} \sum_{\substack{q=u,d \\ \alpha,\beta=e,\mu,\tau}} \left[\bar{\nu}_\alpha \gamma^\mu (1-\gamma^5)\nu_\beta\right] \left(\varepsilon^{qL}_{\alpha\beta}\left[\bar{q}\gamma_\mu(1-\gamma^5)q\right] + \varepsilon^{qR}_{\alpha\beta}\left[\bar{q}\gamma_\mu(1+\gamma^5)q\right]\right),$$

$$\mathcal{L}^{NC}_{\nu Hadron} = -\frac{G_F}{\sqrt{2}} \sum_{\substack{q=u,d \\ \alpha,\beta=e,\mu,\tau}} \left[\bar{\nu}_\alpha \gamma^\mu (1-\gamma^5)\nu_\beta\right] \left(f^{qL}_{\alpha\beta}\left[\bar{q}\gamma_\mu(1-\gamma^5)q\right] + f^{qR}_{\alpha\beta}\left[\bar{q}\gamma_\mu(1+\gamma^5)q\right]\right),$$

$$f^{uL}_{\alpha\alpha} = \rho^{NC}_{\nu N}\left(\frac{1}{2} - \frac{2}{3}\hat{\kappa}_{\nu N}\hat{s}^2_Z\right) + \lambda^{uL} + \varepsilon^{uL}_{\alpha\alpha}$$

$$f^{dL}_{\alpha\alpha} = \rho^{NC}_{\nu N}\left(-\frac{1}{2} + \frac{1}{3}\hat{\kappa}_{\nu N}\hat{s}^2_Z\right) + \lambda^{dL} + \varepsilon^{dL}_{\alpha\alpha}$$

$$f^{uR}_{\alpha\alpha} = \rho^{NC}_{\nu N}\left(-\frac{2}{3}\hat{\kappa}_{\nu N}\hat{s}^2_Z\right) + \lambda^{uR} + \varepsilon^{uR}_{\alpha\alpha}$$

$$f^{dR}_{\alpha\alpha} = \rho^{NC}_{\nu N}\left(\frac{1}{3}\hat{\kappa}_{\nu N}\hat{s}^2_Z\right) + \lambda^{dR} + \varepsilon^{dR}_{\alpha\alpha}$$