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Collective ν flavor transitions in Supernovae:
analytical and numerical aspects

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Outline

- Intro: SN ν self-interactions
- Coupled equations of motion
- Flavor pendulum analogy
- Spectral split/swap
- Discussion of our work(*)
- Conclusions

(*) **G.L. Fogli, E.L., A. Marrone & A. Mirizzi,** "Collective neutrino flavor transitions in supernovae and the role of trajectory averaging" **arXiv:0707.1998 [hep-ph]**

Introduction

Well-known MSW effects can occur in a SN envelope when the ν potential $\lambda = \sqrt{2}\, G_F\, N_e$ is close to osc. frequency $\omega = \Delta m^2/2E$ ($\Delta m^2 = |m^2_3 - m^2_{1,2}|$, $\theta_{13} \neq 0$).

For t~few sec after bounce,

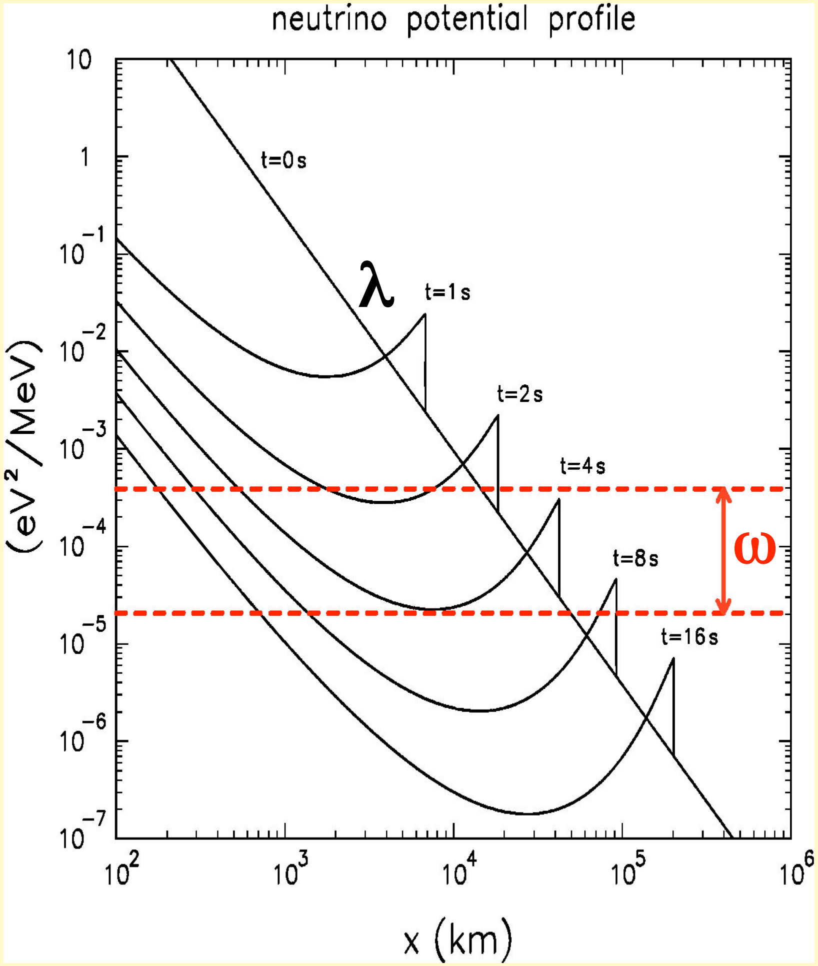
$\lambda \sim \omega$ at **x>>10² km** (large radii).

What about small radii?
Popular wisdom:

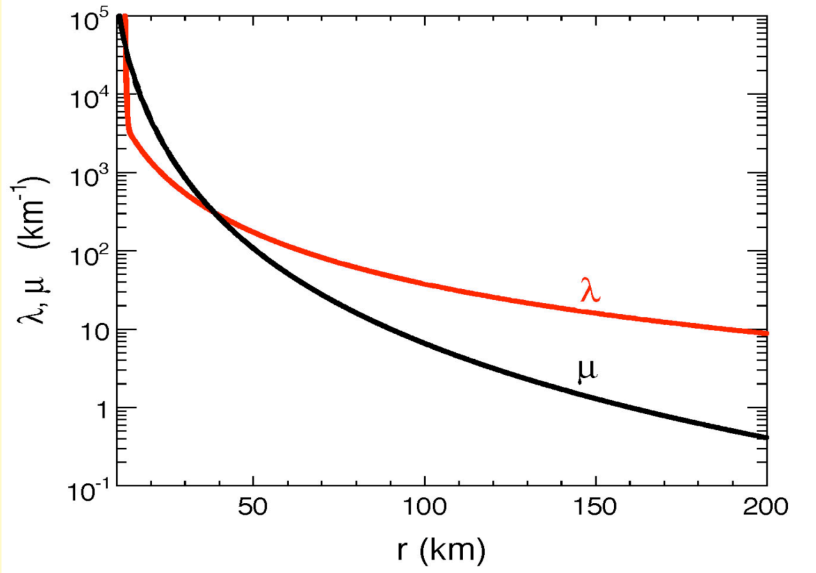
$\lambda >> \omega$ at **x<O(10²) km**,

thus flavor transitions suppressed.

Incorrect!



At small r, neutrino and antineutrino density (n and n̄) high enough to make self-interactions important. Strength: $\mu = \sqrt{2}\, G_F\, (n + \bar{n})$



Angular modulation factor: $(1 - \cos\Theta_{ij})$
If averaged: **"single-angle"** approxim.
Otherwise : **"multi-angle"** (difficult)

Self-interaction effects studied for ~20 y in SN. But, recent boost of interest after new crucial results by Duan, Fuller, Carlson, Qian '05-'06

Lesson: self-interactions (μ) can induce large, non-MSW flavor change at small radii, despite large matter density λ

Recent papers (time-ordered)

[01] Fuller & Qian	astro-ph/0505240	
[02] Duan, Fuller & Qian	astro-ph/0511275	<-- The "synchronized" and "bipolar" regimes
[03] Duan, Fuller, Carlson & Qian	astro-ph/0606616	<-- Large-scale multi-angle calculations
[04] Duan, Fuller, Carlson & Qian	astro-ph/0608050	
[05] Hannestad, Raffelt, Sigl & Wong	astro-ph/0608695	<-- The "flavor pendulum" analogy
[06] Raffelt & Sigl	hep-ph/0701182	
[07] Duan, Fuller, Carlson & Qian	astro-ph/0703776	
[08] Raffelt and Smirnov	hep-ph/0705.1830	<-- The "spectral split"
[09] Esteban, Pastor, Tomas, Raffelt & Sigl	astro-ph/0706.2498	
[10] Duan, Fuller & Qian	astro-ph/0706.4293	
[11] Duan, Fuller, Carlson & Qian	astro-ph/0707.0290	<-- The "spectral split"
[12] Fogli, Lisi, Marrone & Mirizzi	hep-ph/0707.1998	<-- Our work **(this talk)**

Aims of our work:

- Exploration of self-interaction effects for "typical" matter profile with **no MSW effects at small radii** (unlike the shallow profile in [03])
- Test of **robustness of effects** when passing from (simple) single-angle calculations to (difficult but more realistic) multi-angle ones.

Coupled equations of motion (for 2 flavors, e and $x=\mu,\tau$)

Decompose (anti)neutrino density matrix over Pauli matrices to get the "**polarization**" (Bloch) vector **P**. Survival probability P_{ee} related to $\mathbf{P}_Z$.

Discretize over energy spectrum (**N_E bins**), and over angular distribution if multi-angle (**N_Θ bins**) -> Get discrete index (indices), $\mathbf{P}_i$.

Evolution governed by **$6 \times N_E \times N_\Theta$ coupled Bloch equations** of the form:

$$\dot{\mathbf{P}}_i = \mathbf{V}_{\text{ector}}[+\omega,\ \lambda,\ \mu,\ \mathbf{P}_j,\ \overline{\mathbf{P}}_j] \times \mathbf{P}_i$$

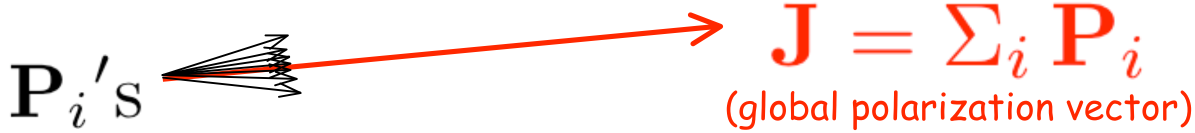
$$\dot{\overline{\mathbf{P}}}_i = \mathbf{V}_{\text{ector}}[-\omega,\ \lambda,\ \mu,\ \mathbf{P}_j,\ \overline{\mathbf{P}}_j] \times \overline{\mathbf{P}}_i$$

vacuum (ω), matter (λ), self-interaction (μ), ij couplings ($\mathbf{P}_j$, $\overline{\mathbf{P}}_j$)

Large, "stiff" set of differential equations

Numerical explorations have systematically shown surprising, non-MSW, "collective" behavior of the polarization vectors $\mathbf{P}_i$'s (and thus of P_{ee}).

Strong couplings make the problem difficult, but also make analytical understanding possible after all ! Key tool of "**near-alignment**":

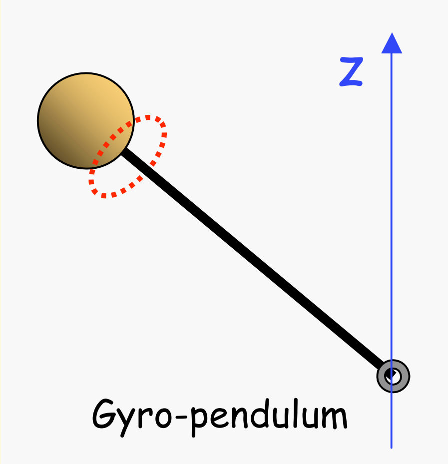


Other items in the theoretical toolbox:

Magnetic analogy ("precession" in generalized magnetic fields)
Co-rotating frames (rotate away precession, matter effects)
Pendulum analogy (a surprising link with classical mechanics)
Adiabaticity (density variations slow w.r.t. oscillation periods)
Lepton number conservation $(\nu_e\overline{\nu}_e \to \nu_x\overline{\nu}_x,\ \text{but}\ \#(\nu_e - \overline{\nu}_e) = \text{const})$
...

The flavor pendulum (Hannestad, Raffelt, Sigl, Wong 2006)

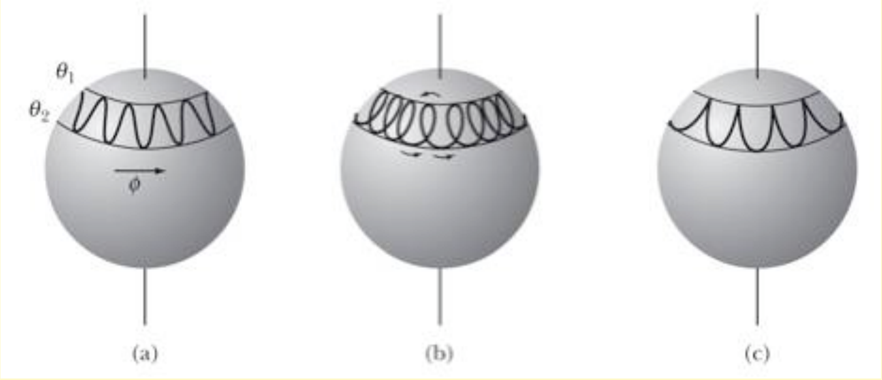
It turns out that a linear combination of the global polarization vectors of neutrinos and antineutrinos obeys the same dynamics of a gyroscopic pendulum (=spherical pendulum with radially spinning mass).



Roughly speaking:

Mass^{-1} ~ (anti)neutrino density

Spin ~ #neutrino - #antineutrino



Generic motion is a combination of

Precession (around z)

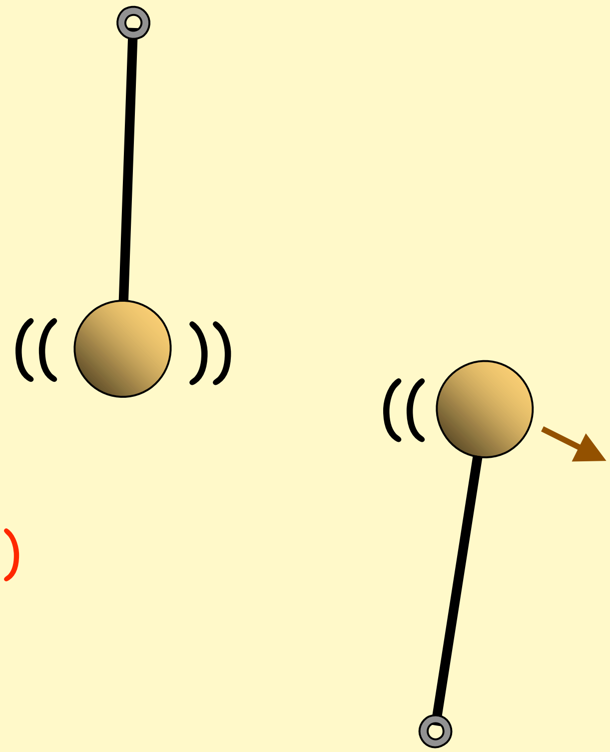
Nutation (along z)

...but with slowly increasing mass!

Neutrino mass hierarchy (and θ_{13}) set initial conditions and fate.

Normal hierarchy:

Pendulum starts in ~downward (stable) position and stays nearby.
No significant flavor change.



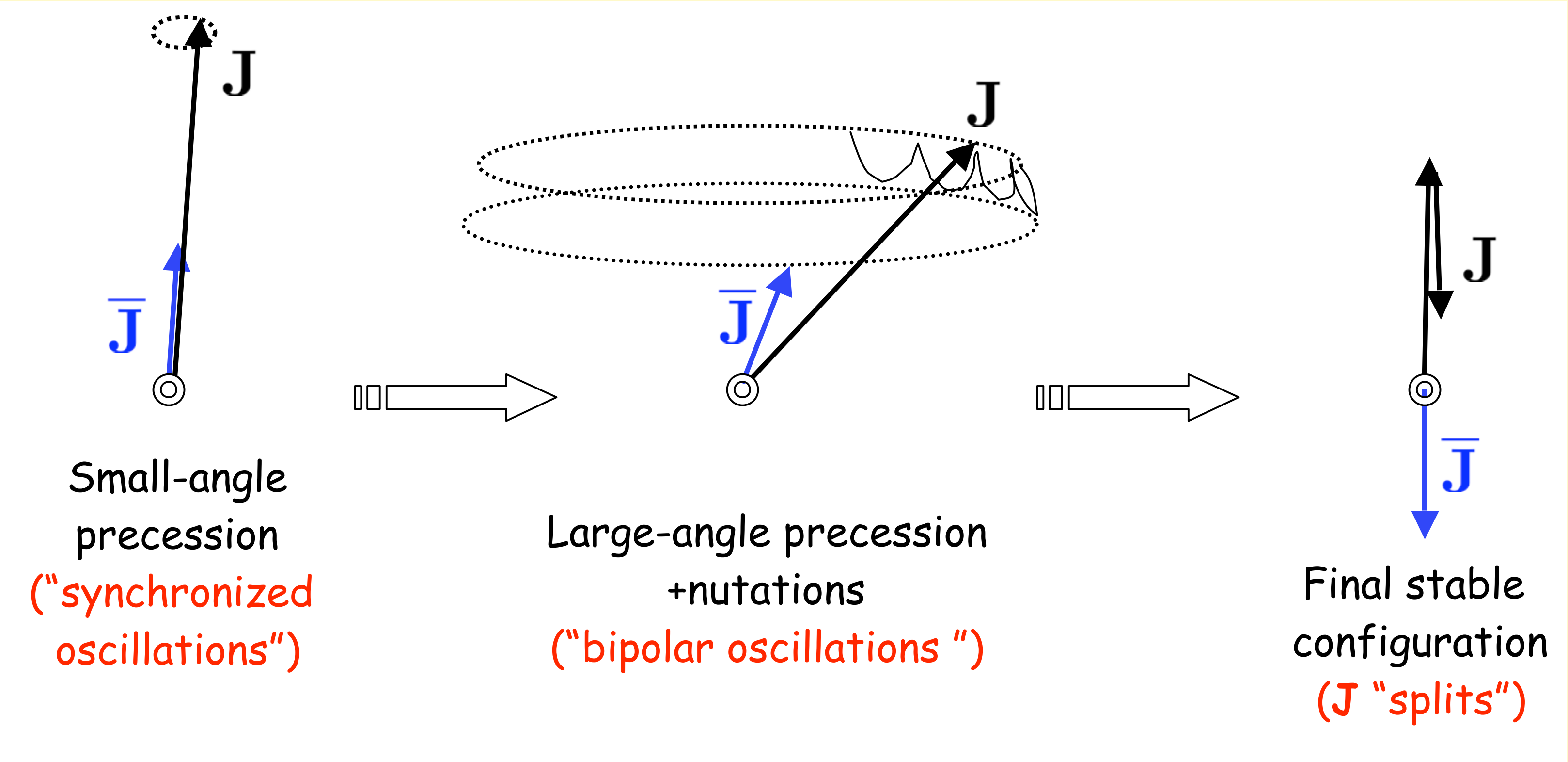
Inverted hierarchy:

Pendulum starts in ~upward (unstable) position and eventually falls down.
Significant flavor changes.

θ_{13} sets initial misalignment with vertical. Specific value not much relevant (provided that $\theta_{13} > 0$).
Only for $\theta_{13} = 0$ exactly, initial conditions are "frozen".

The spectral split (hereafter, inv. hierarchy and θ_{13} >0 assumed)

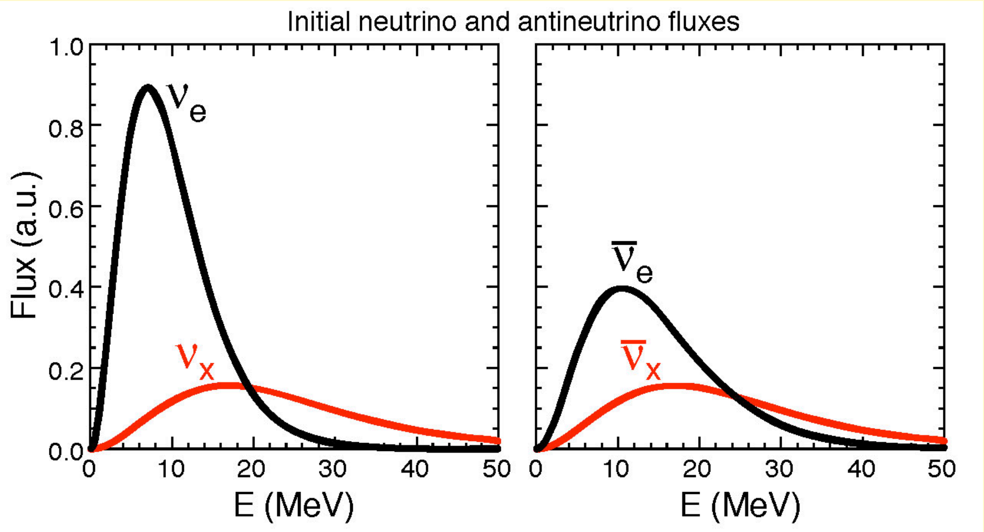
Global polarization vectors (**J** and $\underline{\mathbf{J}}$, with |**J**|>|$\underline{\mathbf{J}}$|) follow pendulum motion as far as near-alignment holds. Eventually $\underline{\mathbf{J}}$ reaches the stable downward position, while **J** can't, to preserve lepton number conservation (~J_Z-$\underline{J}_Z$)



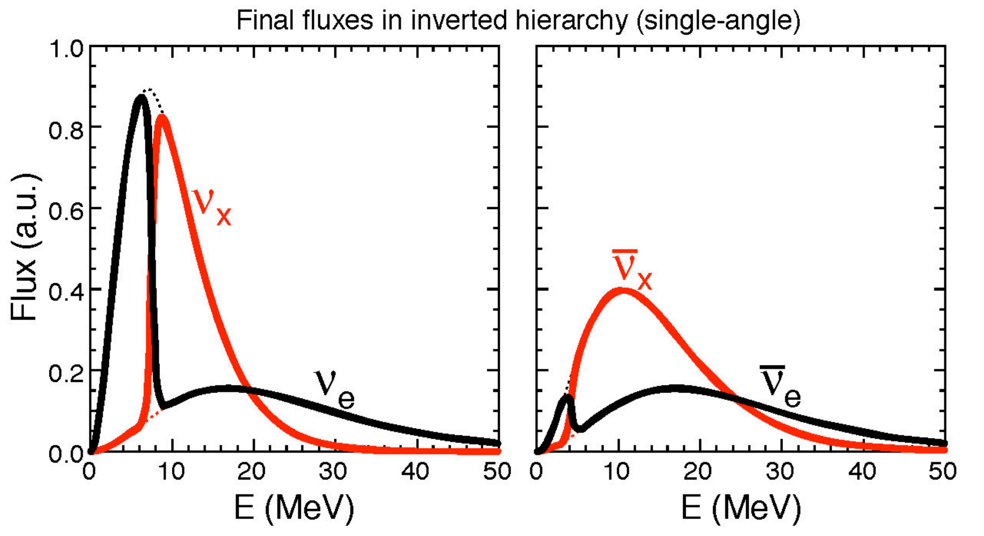
Final state: whole $\underline{\mathbf{J}}$ and **high-E part of J** inverted (**spectral split/swap**)

(Inversion = complete flavor change)

Our results for the spectral split/swap (inv. hier.)



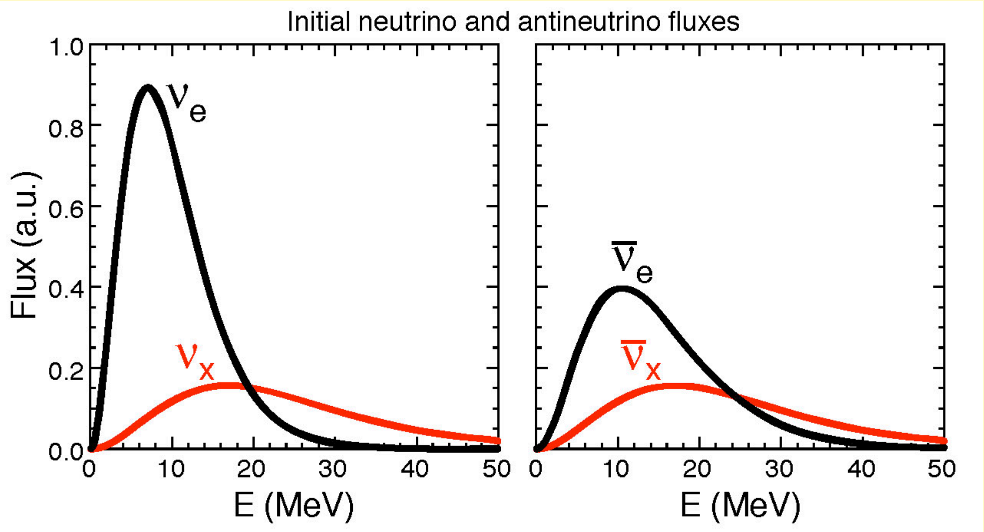
Initial fluxes at the neutrinosphere (r~10 km)



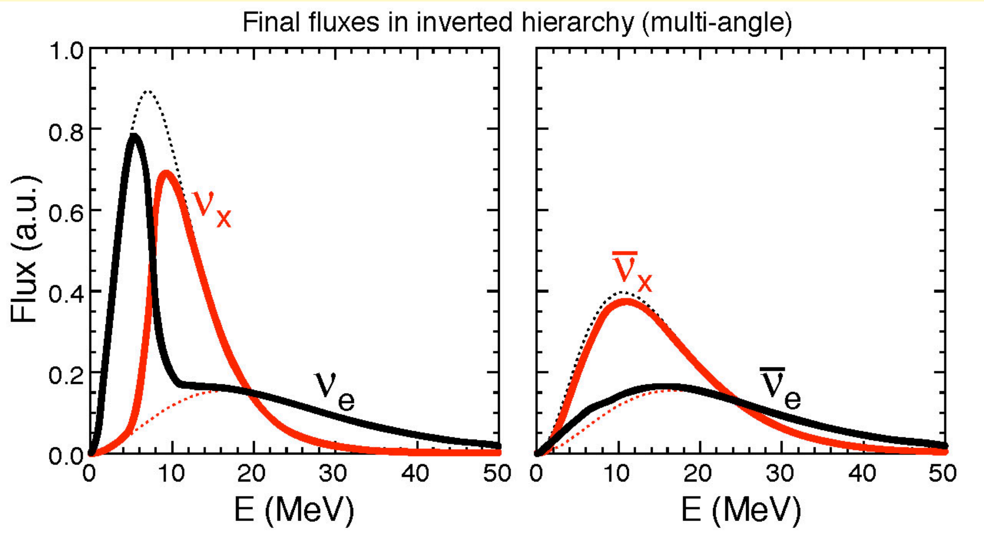
Final fluxes at the end of collective effects (r~200 km)

[Single-angle approximation]

Our results for the spectral split/swap (inv. hier.)



Initial fluxes at the neutrinosphere (r~10 km)



Final fluxes at the end of collective effects (r~200 km)

[Multi-angle calculation, note smearing effect]

Spectral split/swap of neutrino spectra appears to be a robust signature of self-interaction effects in SN for inverse hierarchy

(Not much happens in normal hierarchy.)

It needs nonzero θ_{13} to build up, but specific value of θ_{13} is of little relevance (for definiteness, θ_{13}=0.01 in our work)

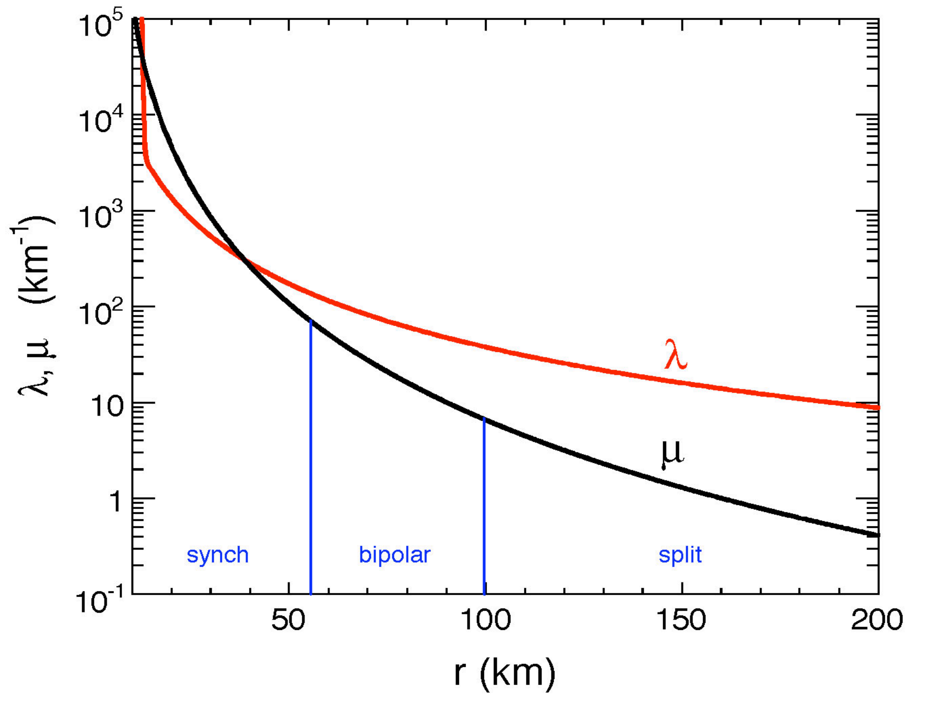
Might be the "ultimate test" of $\theta_{13} > 0$ & of inverted hierarchy

The neutrino splitting energy (~7 MeV in our case) is determined only by lepton number conservation (1 equation in 1 unkown)

"Final" spectra at r~200 km represent the new "initial conditions" for the subsequent MSW evolution (if any) at larger radii

Oscillations between ~10 and ~200 km

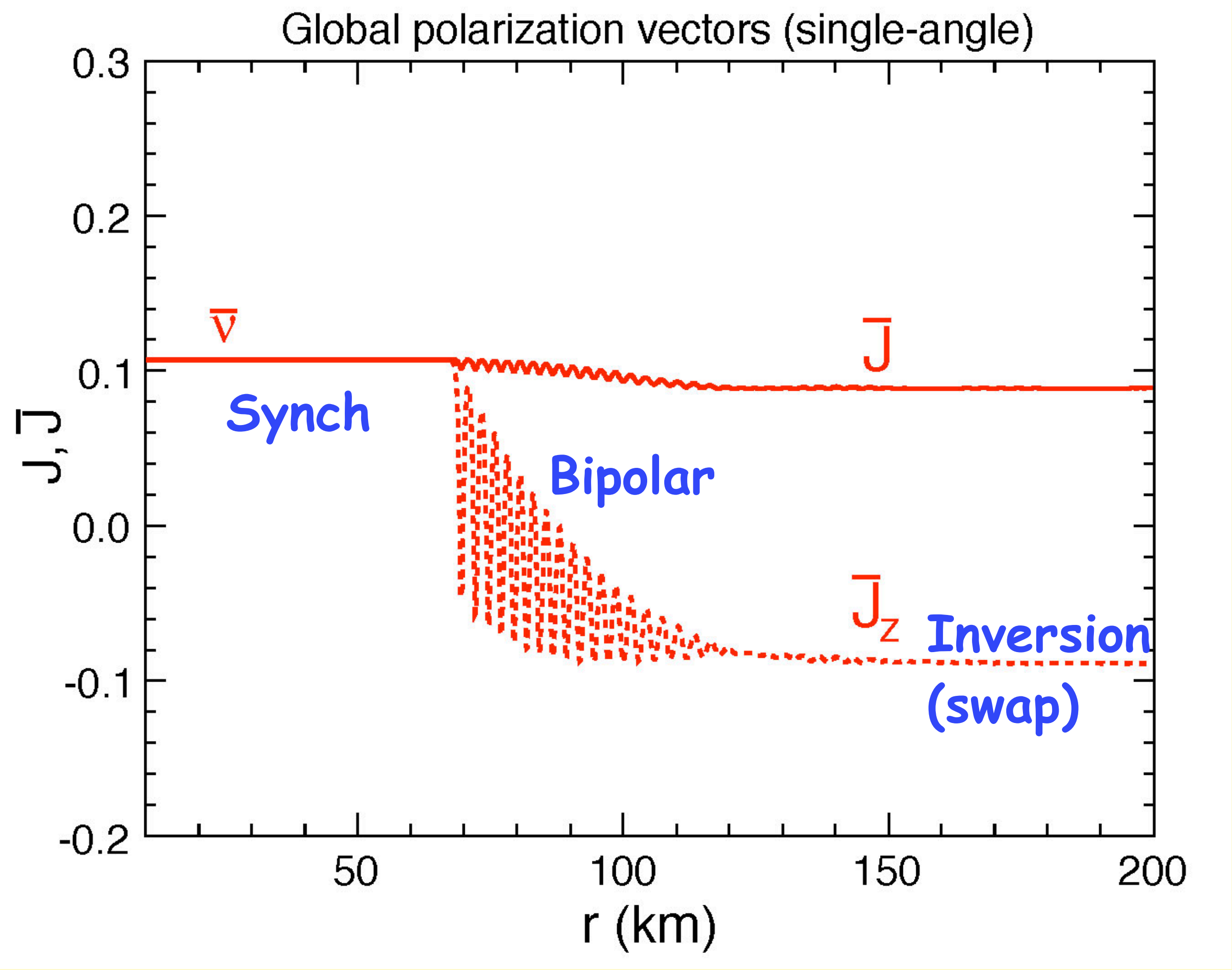
Analytical expectations for characteristic ranges:



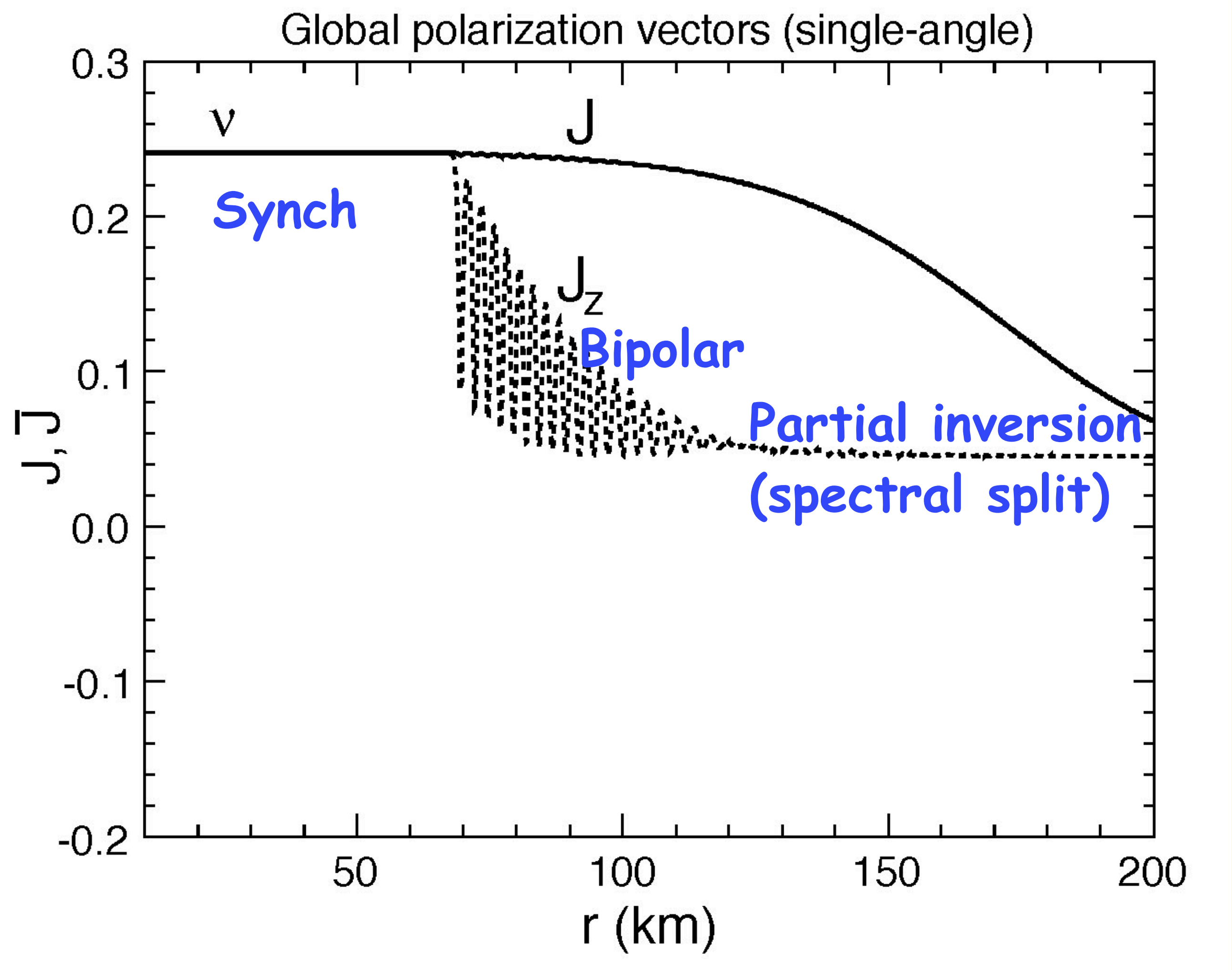
Confirmed by our numerical simulations in single and multi-angle cases.

Main difference between "single-angle" and "multi-angle" results: smearing of bipolar oscillations. Basic features remain robust.

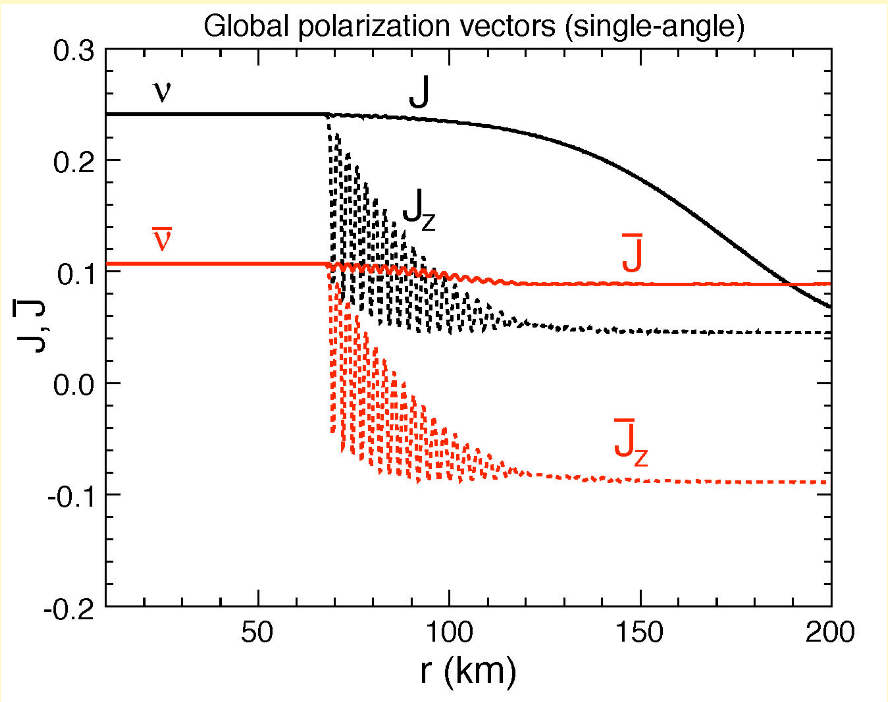
Antineutrinos: numerical results (single-angle)

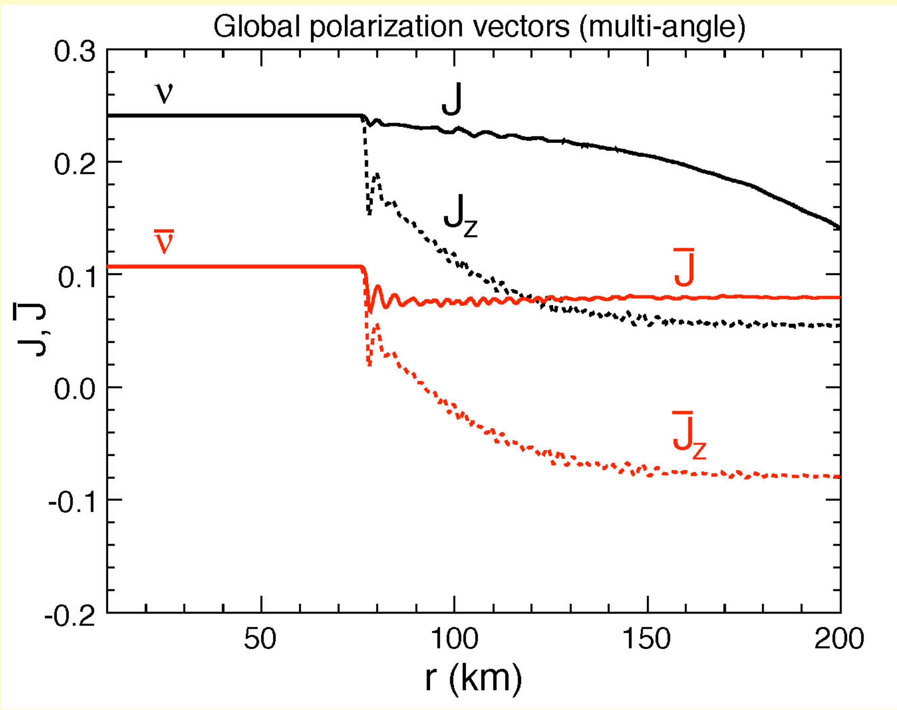


Neutrinos: numerical results (single-angle)



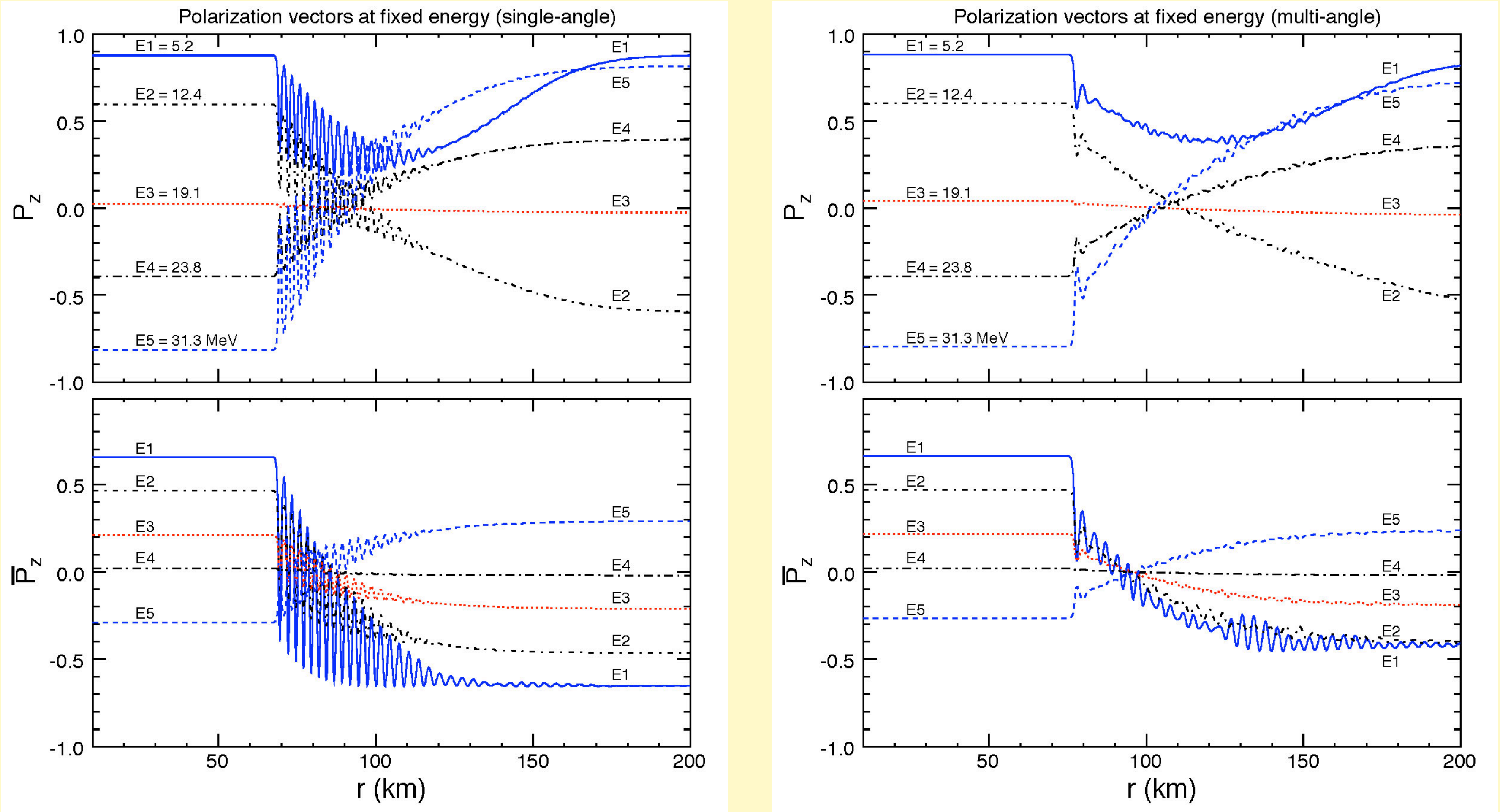
Single-angle vs Multi-angle





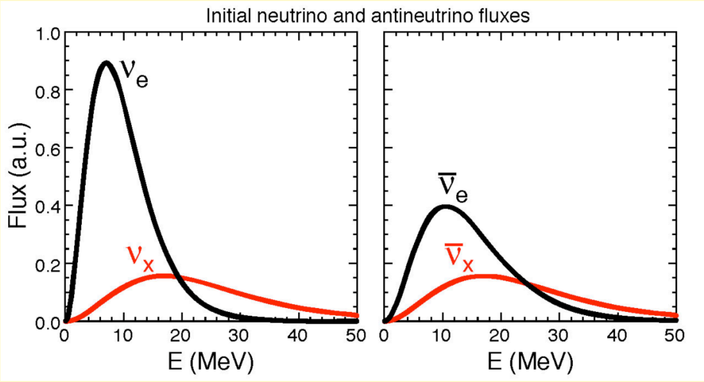
Note smearing of bipolar oscillations.
Other features are qualitatively similar.

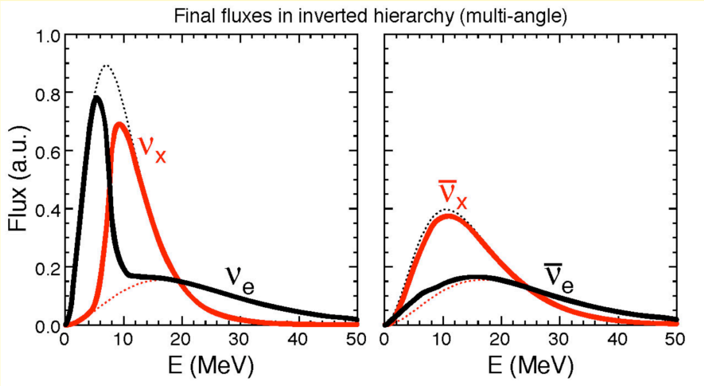
Single-angle vs Multi-angle (individual components P_i)



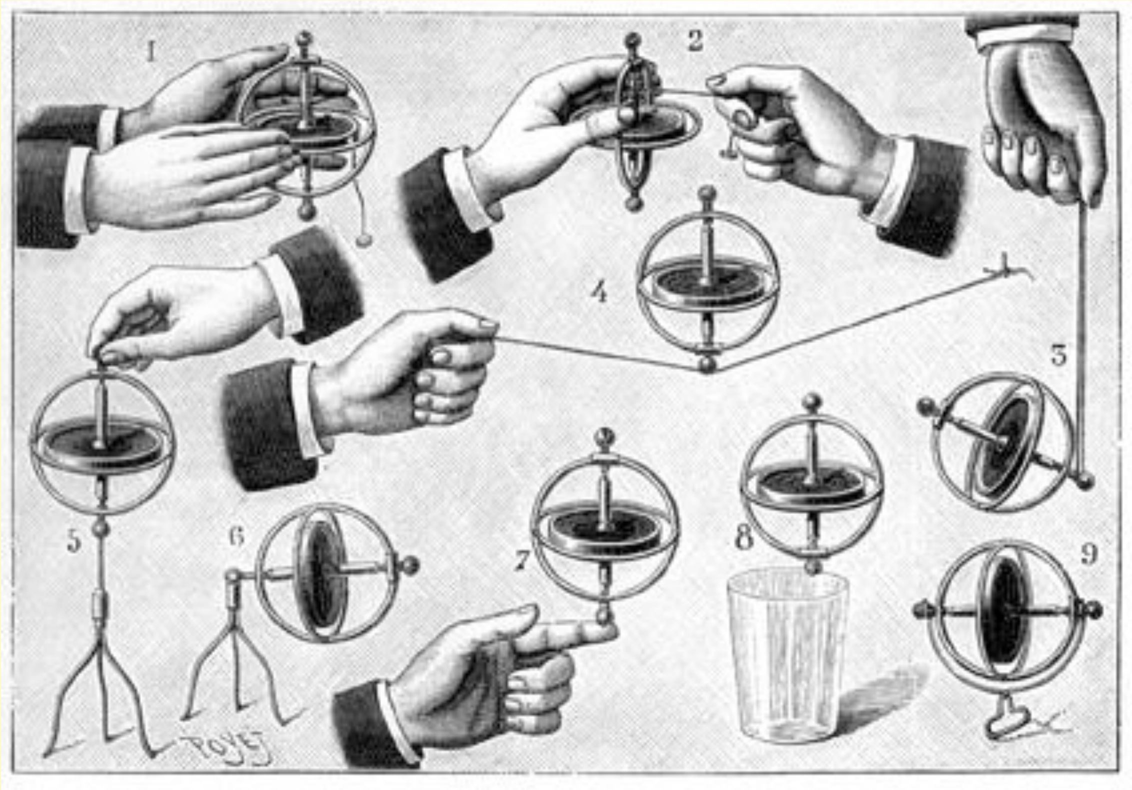
Conclusions

For experimentalists: Spectral split/swap is a robust, observable and well-understood signature of SN neutrino self-interactions in inverted hierarchy (provided that θ_{13} is nonzero)





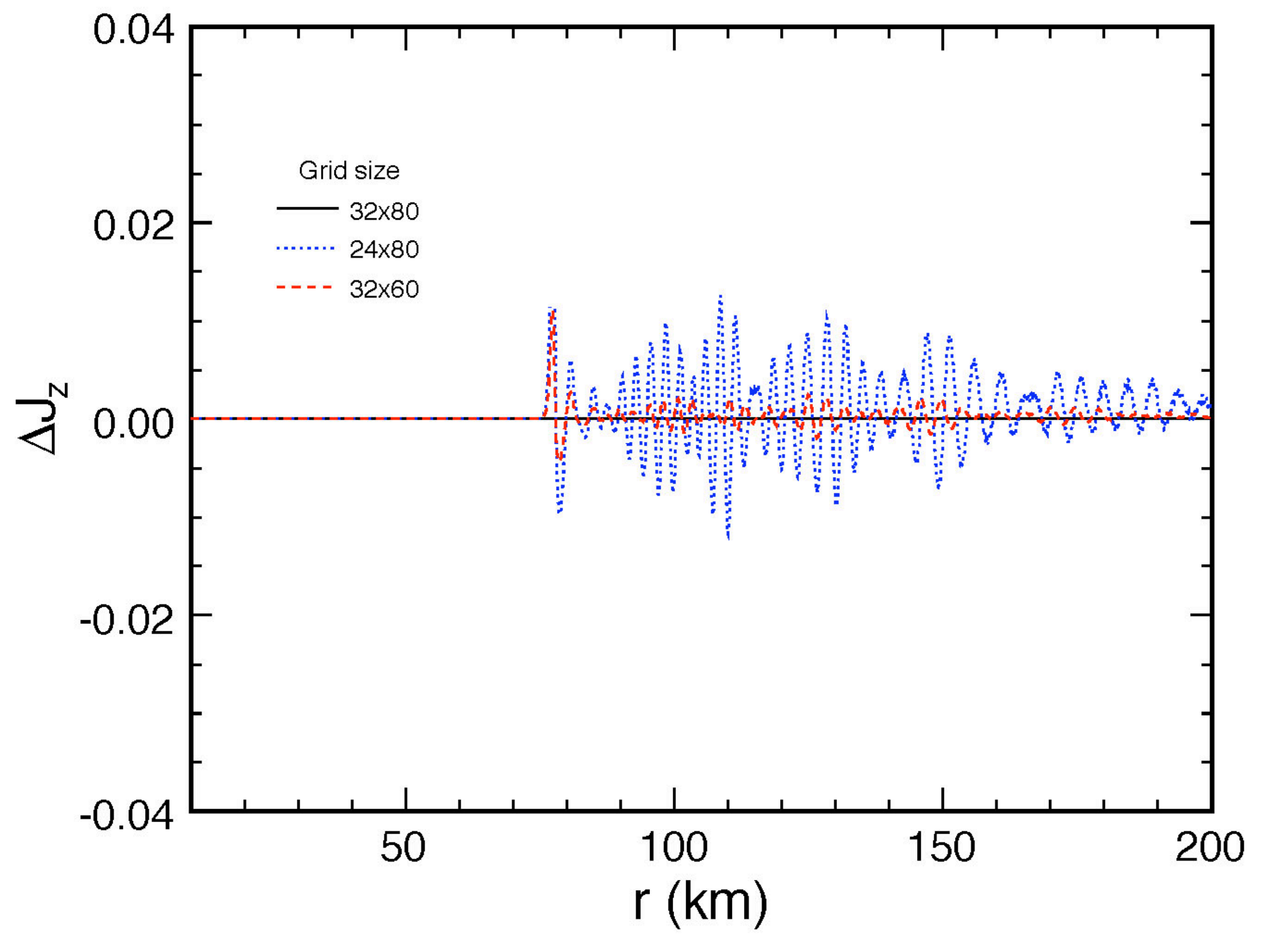
For theorists: Playing with the flavor gyro-pendulum is fun!



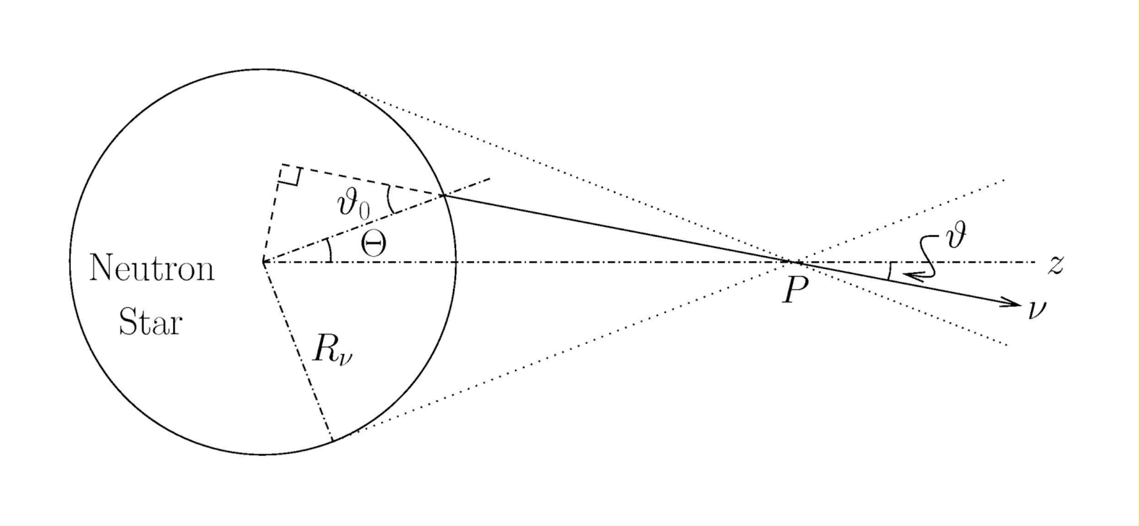
Thank you for your attention.

Backup slides

Test of numerical convergence



Neutrino bulb model: *geometry*



Pauli and Bohr interested in a spinning top