Neutrino Physics Prospects of $(\beta\beta)_{0\nu}$–Decay

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Compelling Evidence for $\nu-$Oscillations: 3-ν mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;
Z. Maki, M. Nakagawa, S. Sakata, 1962;

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ . \qquad (1)$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \qquad (2)$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P.,1980

PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix} \tag{3}$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \tag{4}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.

S.M. Bilenky, J. Hosek, S.T.P.,1980

- $\Delta m^2_\odot \equiv \Delta m^2_{21} \cong 8.0 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.30$, $\cos 2\theta_{12} \gtrsim 0.28$ (2σ),
- $|\Delta m^2_{\rm atm}| \equiv |\Delta m^2_{31}| \cong 2.5 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} < 0.027$ (0.041) 2σ (3σ).

A.Bandyopadhyay, S.Choubey, S.Goswami, S.T.P., D.P.Roy, hep-ph/0406328 (updated);

T. Schwetz, hep-ph/0606060.

- $\sqrt{\Delta m^2_\odot}\,\sin^2\theta_{12} \cong 3.0 \times 10^{-3}$ eV $\;(\pm)\;$ $\sqrt{|\Delta m^2_{\rm atm}|}\,\sin^2\theta_{13} \lesssim 2.2 \times 10^{-3}$ eV;
- $\sqrt{|\Delta m^2_{\rm atm}|} \cong 5 \times 10^{-2}$ eV; $\sqrt{|\Delta m^2_{\rm atm}|}\cos 2\theta_{12} \gtrsim 1.4 \times 10^{-2}$ eV $(\cos 2\theta_{12} \gtrsim 0.28)$
- m_0: $m_0^2 \gg \Delta m^2_\odot, |\Delta m^2_{\rm atm}|$, $m_0 \gtrsim 0.1$ eV
- ${\rm sgn}(\Delta m^2_{\rm atm}) = {\rm sgn}(\Delta m^2_{31})$ not determined

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - NMO, $\;m_3 < m_1 < m_2$ - IMO

- **Majorana phases α_{21}, α_{31}:**
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $|<m>|$ in $(\beta\beta)_{0\nu}$−decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\alpha_{21,31}$!

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of sgn($\Delta m^2_{\rm atm}$), type of $\nu-$ mass spectrum

$$m_1 \ll m_2 \ll m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\rm atm},\ \text{QD};\ m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2\theta_{13}$.

- High precision determination of $\Delta m^2_{\odot}$, $\theta_{\odot}$, $\Delta m^2_{\rm atm}$, θ_{atm}.

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. decays.

• Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

– the origin of the observed patterns of ν-mixing and ν-masses ;

– the physical origin of CPV phases in U_{PMNS} ;

– Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

– Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

– Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

– Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS}?

• Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

$(\beta\beta)_{0\nu}-$Decay Experiments:

- Majorana nature of ν_j
- Type of $\nu-$mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^{3}H β-decay, cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

ν_j- Dirac or Majorana particles, **fundamental problem**

ν_j-Dirac: **conserved lepton charge exists,** $L = L_e + L_\mu + L_\tau$**,** $\nu_j \neq \bar{\nu}_j$

ν_j-Majorana: **no lepton charge is exactly conserved,** $\nu_j \equiv \bar{\nu}_j$

The observed patterns of $\nu-$mixing and of Δm^2_{atm} and $\Delta m^2_\odot$ can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j- Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of $\nu-$ oscillations.

If $\nu_j -$ Majorana particles, U_{PMNS} **contains (3-ν mixing)**

δ-Dirac, α_{21}, α_{31} - Majorana **physical CPV phases**

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- **are not sensitive to the nature of ν_j,**

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker et al., 1987

- **provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.**

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$
$$\mu^- + \text{ (A, Z)} \rightarrow \mu^+ + \text{ (A, Z} - 2)$$

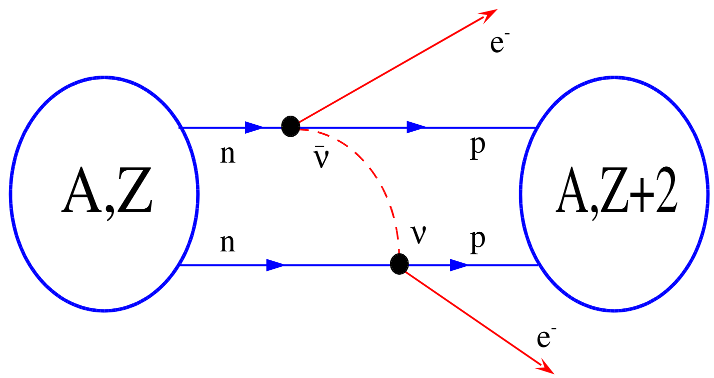
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$\text{(A, Z)} \rightarrow \text{(A, Z} + 2) + e^- + e^-$$

of the even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

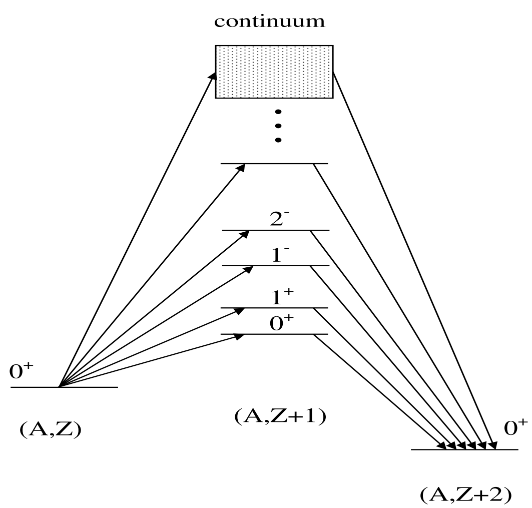
2n from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into 2p of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$$dd \rightarrow uue^-e^-(\bar{\nu}_e\bar{\nu}_e)$$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| = \left|m_1|U_{e1}|^2 + m_2|U_{e2}|^2\ e^{i\alpha_{21}} + m_3|U_{e3}|^2\ e^{i\alpha_{31}}\right|$$

$$= \left|m_1\ c_{12}^2\ c_{13}^2 + m_2\ s_{12}^2\ c_{13}^2\ e^{i\alpha_{21}} + m_3\ s_{13}^2\ e^{i\alpha_{31}}\right|,\ \ \theta_{12} \equiv \theta_\odot,\ \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi,\ \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1,\quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

Best sensitivity: Heidelberg-Moscow ^{76}Ge experiment.

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|{<}m{>}| = (0.1 - 0.9)$ eV (99.73% C.L.).

IGEX ^{76}Ge: $|{<}m{>}| < (0.33 - 1.35)$ eV (90% C.L.).

Taking data - NEMO3 (^{82}Se, ^{100}Mo), CUORICINO (^{130}Te):

$|{<}m{>}|$ <(0.7–1.2) eV, $|{<}m{>}|$ <(0.18–0.90) eV (90% C.L.).

Large number of projects: $|{<}m{>}| \sim (0.01 - 0.05)$ eV

CUORE - ^{130}Te,
GERDA - ^{76}Ge,
SuperNEMO - ^{82}Se,
EXO - ^{136}Xe,
MAJORANA - ^{76}Ge,
MOON - ^{100}Mo,
CANDLES - ^{48}Ca,
XMASS - ^{136}Xe.

$|{<}m{>}|$: m_j, $\theta_\odot \equiv \theta_{12}$, θ_{13}, $\alpha_{21,31}$

$m_{1,2,3}$ - in terms of min(m_j), $\Delta m^2_{\rm atm}$, $\Delta m^2_\odot$

S.T.P., A.Yu. Smirnov, 1994

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$\Delta m^2_\odot \equiv \Delta m^2_{21}, \quad m_2 = \sqrt{m_1^2 + \Delta m^2_\odot},$$

while either

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{\rm atm}}, \quad \text{normal mass ordering}, \quad \text{or}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad m_1 = \sqrt{m_3^2 + |\Delta m^2_{\rm atm}| - \Delta m^2_\odot}, \quad \text{inverted mass ordering}$$

The neutrino mass spectrum –

Normal hierarchical (NH) if $m_1 << m_2 << m_3$,

Inverted hierarchical (IH) if $m_3 << m_1 \cong m_2$,

Quasi-degenerate (QD) if $m_1 \cong m_2 \cong m_3 = m$, $m_j^2 >> |\Delta m^2_{\rm atm}|$; $m_j \gtrsim 0.1$ eV

Given $|\Delta m^2_{\rm atm}|$, $\Delta m^2_\odot$, $\theta_\odot$, θ_{13},

$$|{<}m{>}| = |{<}m{>}|\ (\mathrm{m_{min}}, \alpha_{21}, \alpha_{31}; \mathrm{S}),\ \mathrm{S} = \mathrm{NO(NH), IO(IH)}.$$

$$A(\beta\beta)_{0\nu} \sim \, < m > \; \text{M(A,Z)}, \qquad \text{M(A,Z) - NME},$$

$$\left| < m > \right| \; \cong \left| \sqrt{\Delta m^2_\odot}\, \sin^2\theta_{12} e^{i\alpha} + \sqrt{\Delta m^2_{31}}\, \sin^2\theta_{13} e^{i\beta} \right| \; , \; m_1 \ll m_2 \ll m_3 \; \text{(NH)},$$

$$\left| < m > \right| \; \cong \sqrt{m_3^2 + \Delta m^2_{23}} \left| \cos^2\theta_{12} + e^{i\alpha} \; \sin^2\theta_{12} \right| \; , \; m_3 < (\ll) m_1 < m_2 \; \text{(IH)},$$

$$\left| < m > \right| \; \cong m \left| \cos^2\theta_{12} + e^{i\alpha} \; \sin^2\theta_{12} \right| \; , \; m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV} \;\; \text{(QD)},$$

$\theta_{12} \equiv \theta_\odot$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta = 0, \pm\pi$;

$$\left| < m > \right| \; \lesssim 5 \times 10^{-3} \text{ eV, NH};$$

$$\sqrt{\Delta m^2_{23}} \cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim \; \left| < m > \right| \; \lesssim \sqrt{\Delta m^2_{23}} \cong 0.055 \text{ eV}, \quad \text{IH};$$

$$m \cos 2\theta_{12} \lesssim \; \left| < m > \right| \; \lesssim m, \; m \gtrsim 0.10 \text{ eV}, \;\; \text{QD} \; .$$

$\nu_\odot$, Δm^2_{atm}, CHOOZ Data:

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{6}$, $\qquad \theta_{23} = \theta_{\text{atm}} \cong \frac{\pi}{4}$, $\qquad \theta_{13} < \frac{\pi}{12}$

$$U_{\text{PMNS}} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & \epsilon \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} .$$

Very different from the CKM-matrix!

- $\cos\theta_{12} \cong \cos(\frac{\pi}{4} - \frac{\pi}{12}) = \frac{1}{\sqrt{2}}(1+\lambda)$, $\quad \sin\theta_{12} \cong \frac{1}{\sqrt{2}}(1-\lambda)$,
- $\lambda \cong (0.20 - 0.25)$: $\quad \theta_\odot + \theta_c = \pi/4$?

Natural Possibility:

$$U = U_{\text{lep}}^{\dagger}(\lambda)\; U_{\text{bim(tri)}}$$

with

$$U_{\text{bim}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad U_{\text{tri}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}.$$

- $U_{\text{lep}}^{\dagger}(\lambda)$ - from diagonalization of the l^- mass matrix,
- $U_{\text{bim(tri)}}$ - from diagonalization of the $\nu-$mass matrix

Further, $\Delta m^2_{\odot} \ll |\Delta m^2_{\text{atm}}|$.

- U_{bim} can be associated with a symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

- $U_{\text{bim(tri)}}$ can be associated with a $\mu - \tau$ symmetry of M_ν

T. Fukuyama, H. Nishiura, 1997; R.N. Mohapatra, S. Nussinov, 1999;...

These symmetries cannot be exact.

The Case of CP Nonconservation

$M_{\text{lep}} = U_L^\dagger \, m_{\text{lep}}^{\text{diag}} \, U_R$, $M_\nu = U_\nu^T \, m_\nu^{\text{diag}} \, U_\nu$; $\quad U = e^{i\Phi} \, P \, \tilde{U} \, Q$

$$U_{\text{PMNS}} = U_{\text{L}}^\dagger \, U_\nu = \tilde{U}_{\text{lep}}^\dagger \; P_\nu \; \tilde{U}_\nu \; Q_\nu$$

- $\tilde{U}_{\text{lep}}$, $\tilde{U}_\nu$ - CKM-like: (3+3) angles, (1+1) CPVP
- $P_\nu = \text{diag}(1, e^{i\phi}, e^{i\omega})$, $Q_\nu = \text{diag}(1, e^{i\rho}, e^{i\sigma})$: 4 CPVP

U_{PMNS}: 3 angles, 3 CPVP

$\tilde{U}_{\text{lep}}^\dagger \; P_\nu \; \tilde{U}_\nu \; Q_\nu$: 6 angles, 6 CPVP; textures, symmetries

$$U_\nu = P\,\tilde{U}_\nu\,Q = \mathrm{diag}(1, e^{i\phi}, e^{i\omega})\,\tilde{U}_\nu\,\mathrm{diag}(1, e^{i\sigma}, e^{i\tau})$$

$$= P\,O_{23}(\theta^\nu_{23})\,U_{13}(\theta^\nu_{13}\,, \xi)\,O_{12}(\theta^\nu_{12})\,Q$$

$$= P \begin{pmatrix} c^\nu_{12}\,c^\nu_{13} & s^\nu_{12}\,c^\nu_{13} & s^\nu_{13}\,e^{-i\xi} \\ -s^\nu_{12}\,c^\nu_{23} - c^\nu_{12}\,s^\nu_{23}\,s^\nu_{13}\,e^{i\xi} & c^\nu_{12}\,c^\nu_{23} - s^\nu_{12}\,s^\nu_{23}\,s^\nu_{13}\,e^{i\xi} & s^\nu_{23}\,c^\nu_{13} \\ s^\nu_{12}\,s^\nu_{23} - c^\nu_{12}\,c^\nu_{23}\,s^\nu_{13}\,e^{i\xi} & -c^\nu_{12}\,s^\nu_{23} - s^\nu_{12}\,c^\nu_{23}\,s^\nu_{13}\,e^{i\xi} & c^\nu_{23}\,c^\nu_{13} \end{pmatrix} Q$$

$$\tilde{U}_\ell = O_{23}(\theta^\ell_{23})\,U_{13}(\theta^\ell_{13}, \psi)\,O_{12}(\theta^\ell_{12})$$

$$= \begin{pmatrix} c^\ell_{12}\,c^\ell_{13} & s^\ell_{12}\,c^\ell_{13} & s^\ell_{13}\,e^{-i\psi} \\ -s^\ell_{12}\,c^\ell_{23} - c^\ell_{12}\,s^\ell_{23}\,s^\ell_{13}\,e^{i\psi} & c^\ell_{12}\,c^\ell_{23} - s^\ell_{12}\,s^\ell_{23}\,s^\ell_{13}\,e^{i\psi} & s^\ell_{23}\,c^\ell_{13} \\ s^\ell_{12}\,s^\ell_{23} - c^\ell_{12}\,c^\ell_{23}\,s^\ell_{13}\,e^{i\psi} & -c^\ell_{12}\,s^\ell_{23} - s^\ell_{12}\,c^\ell_{23}\,s^\ell_{13}\,e^{i\psi} & c^\ell_{23}\,c^\ell_{13} \end{pmatrix},$$

$$U_{13}(\theta, \kappa) = \begin{pmatrix} \cos\theta & 0 & \sin\theta\,e^{-i\kappa} \\ 0 & 1 & 0 \\ -\sin\theta\,e^{i\kappa} & 0 & \cos\theta \end{pmatrix}$$

Suppose $\tilde{U}_\nu$ - bimaximal (real) and arises from

$$M_\nu = \frac{m}{\sqrt{2}} \left(\begin{array}{ccc} 0 & e^{-i\alpha'} & e^{-i\beta'} \\ e^{-i\alpha'} & 0 & \epsilon\ e^{-i\gamma'} \\ e^{-i\beta'} & \epsilon\ e^{-i\gamma'} & 0 \end{array} \right) ,$$

α', β', γ' - phases, $\epsilon \ll 1$

$\Delta m^2_{\rm atm} \cong m^2$, $\Delta m^2_\odot \cong \sqrt{2}\epsilon \Delta m^2_{\rm atm}$, $\epsilon \sim 0.025$, IH $\nu-$ masses

In the limit $\epsilon = 0$ and $U_{\rm lep} = \mathbf{1}$,

$L' = L_e - L_\mu - L_\tau$ is conserved.

For $U_{\rm lep} \neq \mathbf{1}$, $(\alpha' - \gamma')$, $(\beta' - \gamma')$ physical CPVP,

$Q_\nu = \mathbf{1}$, $P_\nu = {\rm diag}(1, e^{i(\beta'-\gamma')}, e^{i(\alpha'-\gamma')})$

$U_{\rm PMNS} = \tilde{U}^\dagger_{\rm lep} P_\nu U_{\rm bimax}$: 3 angles, 3 CPVP

In general, Dirac and Majorana CPV phases are independent.

However, for all $\sin^\ell \theta_{ij} \equiv \lambda_{ij} \lesssim \lambda$ small, in the model we are considering and to leading order in λ,

$$|{<}m{>}| \ \cong \sqrt{|\Delta m^2_{\rm atm}|}\ |\cos 2\theta_\odot + i\, 8\, J_{CP}|\ .$$

P. Frampton, S.T.P., W. Rodejohann, 2004

Rephasing Invariants Associated with CPVP

Dirac phase δ:

$$J_{CP} = \text{Im}\left\{U_{e1}\,U_{\mu 2}\,U^*_{e2}\,U^*_{\mu 1}\right\}\ .$$

C. Jarlskog, 1985

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31}:

$$S_1 = \text{Im}\{U_{e1}U^*_{e3}\}\,,\quad S_2 = \text{Im}\{U_{e2}U^*_{e3}\}\qquad \text{(not unique)}$$

J.F. Nieves and P. Pal, 1987, 2001
G.C. Branco et al., 1986
J.A. Aguilar-Saavedra and G.C. Branco, 2000

S_1, S_2 appear in $|{<}m{>}|$ in $(\beta\beta)_{0\nu}$-decay.

In general, J_{CP}, S_1 and S_2 are independent.

However, for, e.g., all $\sin^\ell\theta_{ij} \equiv \lambda_{ij} \lesssim \lambda$ small, in the model we are considering and to leading order in λ,

$$J_{CP} \simeq \frac{S_1}{2\sqrt{2}} \simeq \frac{S_2}{2\sqrt{2}}\ ,$$

and

$$|{<}m{>}|\ \cong \sqrt{|\Delta m^2_{\text{atm}}|}\ |\cos 2\theta_\odot + i\,8\,J_{CP}|\ .$$

P. Frampton, S.T.P., W. Rodejohann, 2004

Suppose further that M_ν has $\mu - \tau$ symmetry:

$$U_\nu = P\,\tilde{U}_\nu\,Q = \mathrm{diag}(1, e^{i\phi}, e^{i\omega})\,\tilde{U}_\nu\,\mathrm{diag}(1, e^{i\sigma}, e^{i\tau})$$

$$= P\,O_{23}(\theta^\nu_{23} = -\pi/4)\,U_{13}(\theta^\nu_{13} = 0\,, \xi)\,O_{12}(\theta^\nu_{12})\,Q$$

$$= P\,O_{23}(-\pi/4)\,O_{12}(\theta^\nu_{12})\,Q$$

$$\tilde{U}_\ell = O_{23}(\theta^\ell_{23} = 0)\,U_{13}(\theta^\ell_{13} = 0, \psi)\,O_{12}(\theta^\ell_{12}) = O_{12}(\theta^\ell_{12})$$

Now

$$U_{\mathrm{PMNS}} = \tilde{U}^\dagger_{\mathrm{lep}}\,U_\nu = O^T_{12}(\theta^\ell_{12})\,P\,O_{23}(-\pi/4)\,O_{12}(\theta^\nu_{12})\,Q$$

$$= \tilde{P}\,O_{12}(-\theta^\ell_{12})\,\mathrm{diag}(e^{-i\phi}, 1, 1)\,O_{23}(-\pi/4)\,O_{12}(\theta^\nu_{12})\,Q$$

$$\tilde{U}_\nu = O_{12}(\tilde{\theta}_{12})\,\mathrm{diag}(e^{-i\delta'}, 1, 1)\,O_{23}(\tilde{\theta}_{23})\,O_{12}(\theta'_{12})$$

$$= \begin{pmatrix} c'_{12}\,\tilde{c}_{12}\,e^{-i\delta'} - \tilde{c}_{23}\,s'_{12}\,\tilde{s}_{12} & \tilde{c}_{12}\,s'_{12}\,e^{-i\delta'} + c'_{12}\,\tilde{c}_{23}\,\tilde{s}_{12} & \tilde{s}_{12}\,\tilde{s}_{23} \\ -\tilde{c}_{12}\,\tilde{c}_{23}\,s'_{12} - c'_{12}\,\tilde{s}_{12}\,e^{-i\delta'} & c'_{12}\,\tilde{c}_{12}\,\tilde{c}_{23} - s'_{12}\,\tilde{s}_{12}\,e^{-i\delta'} & \tilde{c}_{12}\,\tilde{s}_{23} \\ s'_{12}\,\tilde{s}_{23} & -c'_{12}\,\tilde{s}_{23} & \tilde{c}_{23} \end{pmatrix}\,.$$

For $\sin^{\ell}\theta_{12} \equiv \lambda_{12}$ small, to leading order in λ,

$$\sin^2\theta_{12} = \sin^2\theta^{\nu}_{12} - \sin 2\theta^{\nu}_{12}\, |U_{e3}|\, \cos\phi\ ,$$

ϕ is the Dirac CPV phase,

$$\sin^2\theta_{12} = \sin^2\theta^{\nu}_{12} \pm \sqrt{|U_{e3}|^2\, \sin^2 2\theta^{\nu}_{12} - 16\, J^2_{\rm CP}}\ ,$$

K. Hochmuth, S.T.P., W. Rodejohann, 2007

$$|<m>|\ \cong \sqrt{|\Delta m^2_{\rm atm}|}\, \sqrt{1 - \sin^2 2\theta_{\odot} \sin^2\sigma + 8\, J_{CP} \sin 2\sigma}\,,\quad \text{IH spectrum}\,,$$

$\sigma \equiv \alpha \equiv \alpha_{21}$ is Majorana CPV phase.

L. Everet, S.T.P., 2007

Normal Hierarchical Spectrum, $m_1 \ll m_2 \ll m_3$:

$$m_2 \cong \sqrt{\Delta m^2_\odot} \cong (8.4 - 9.4) \times 10^{-3}\ \text{eV} \quad (3\sigma),$$

$$m_3 \cong \sqrt{\Delta m^2_{\text{atm}}} \cong (4.4 - 5.5) \times 10^{-2}\ \text{eV} \quad (3\sigma).$$

Inverted Hierarchical Spectrum, $m_3 \ll m_1 \cong m_2$:

$$m_{1,2} \cong \sqrt{|\Delta m^2_{\text{atm}}|} \cong (4.4 - 5.5) \times 10^{-2}\ \text{eV}\ .$$

Quasi-Degenerate Spectrum, $m_1 \cong m_2 \cong m_3 \equiv m$:

$$m^2_{1,2,3} >> |\Delta m^2_{\text{atm}}|\ .$$

Using $|\Delta m^2_{\text{atm}}|$ and $\Delta m^2_\odot$ inferred from the data one has

Normal (Inverted) Hierarchical Spectrum for

$$m_1 << 0.02\ \text{eV} \ \ (m_3 < 0.02\ \text{eV})\ ;$$

Spectrum with Partial Hierarchy for

$$0.02\ \text{eV} \lesssim m_{1(3)} \lesssim 0.20\ \text{eV}\ ;$$

Quasi-Degenerate Spectrum for

$$m_{1,2,3} \gtrsim 0.10\ \text{eV}\ .$$

Solar neutrino and KamLAND data:

$\cos 2\theta_\odot = 0.0$ excluded at > 6 s.d.

Best fit value: $\cos 2\theta_\odot \simeq 0.40$

$\cos 2\theta_\odot \gtrsim 0.28$, 95% C.L.

Normal hierarchical spectrum:

$$(|<m>|\ )_{\text{max}} \lesssim 0.005 \text{ eV}$$

Inverted hierarchical spectrum:

$$(|<m>|\ )_{\text{min}} \simeq \sqrt{|\Delta m^2_{\text{atm}}|} \cos 2\theta_\odot \cos^2\theta_{13} \gtrsim 0.01 \text{ eV}$$

$$(|<m>|\ )_{\text{max}} \simeq \sqrt{|\Delta m^2_{\text{atm}}|} \cos^2\theta_{13} \lesssim 0.055 \text{ eV}$$

Quasi-degenerate spectrum:

$$(|<m>|\ )_{\text{min}} \simeq m\ (\cos 2\theta_\odot \cos^2\theta_{13} - \sin^2\theta_{13}) \gtrsim 0.03 \text{ eV}$$

Normal Hierarchical ν−Mass Spectrum

$$m_1 \ll m_2 \ll m_3.$$

This implies:

$$m_2 \simeq \sqrt{\Delta m^2_\odot}, \qquad m_3 \simeq \sqrt{\Delta m^2_{\rm atm}}\ .$$

One has

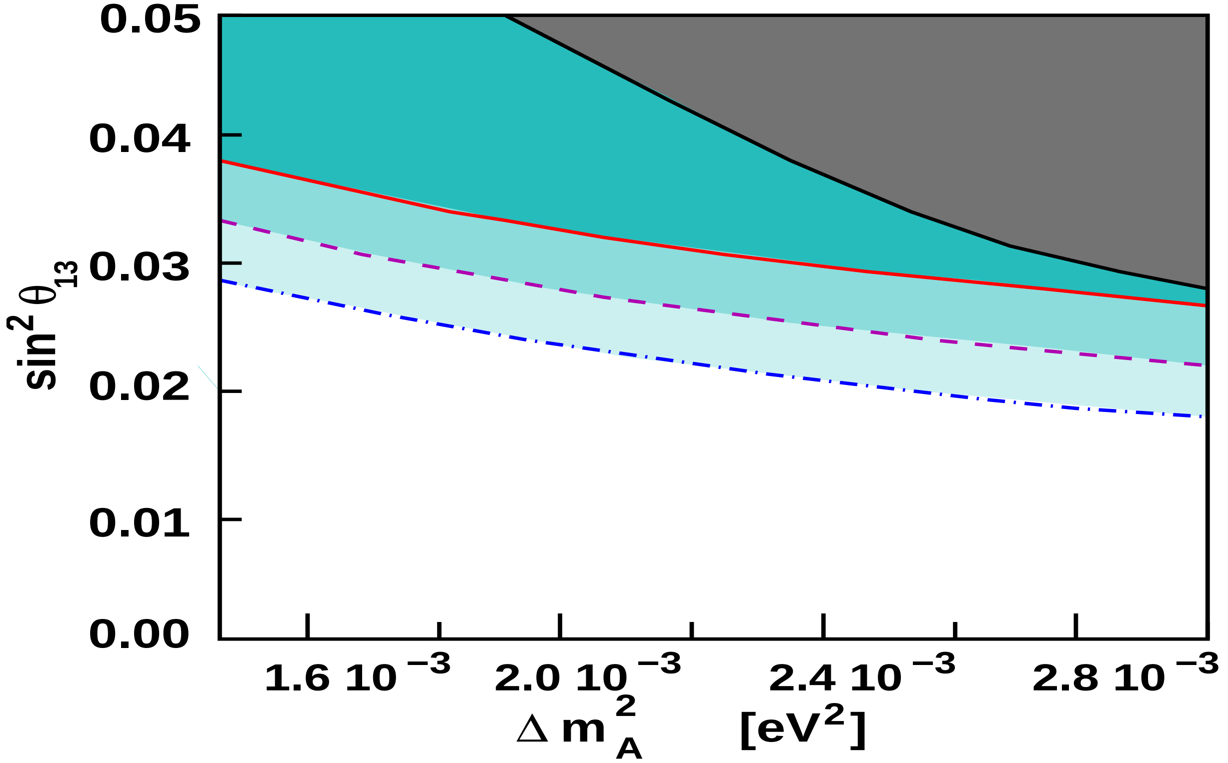
$$|<m>| \ = \left|(m_1 \cos^2\theta_\odot + \sqrt{m_1^2 + \Delta m^2_\odot}\,\sin^2\theta_\odot)(1 - |U_{\rm e3}|^2)e^{i\alpha_{21}}\right.$$

$$\left. + \sqrt{m_1^2 + \Delta m^2_{\rm atm}}|U_{\rm e3}|^2 e^{i\alpha_{31}}\right|$$

$$\simeq \left|\sqrt{\Delta m^2_\odot}(1 - |U_{\rm e3}|^2)\sin^2\theta_\odot + \sqrt{\Delta m^2_{\rm atm}}|U_{\rm e3}|^2 e^{i(\alpha_{31}-\alpha_{21})}\right|$$

Even if $m_1 = 0$, $|<m>|$ depends on $\alpha_{32} = \alpha_{31} - \alpha_{21}$.

$|<m>| \ \lesssim 6 \times 10^{-3}$ eV at 3σ; at 2σ:
$\sqrt{\Delta m^2_{\rm atm}}|U_{\rm e3}|^2 \lesssim 1.5$ meV, $\sqrt{\Delta m^2_\odot}\sin^2\theta_\odot \cong (2.1 - 3.2)$ meV,
$|<m>| \ \gtrsim 0.6$ meV.



S. Pascoli, S.T.P., 2007

$1\sigma(\sin^2\theta_{13}) = 0.004$; $1\sigma(\Delta m^2_{\odot}) = 3.3\%$, $1\sigma(\sin^2\theta_{\odot}) = 4\%$, $1\sigma(|\Delta m^2_{\rm atm}|) = 4\%$

Inverted Hierarchical $\nu-$Mass Spectrum

$$m_3 \ll m_1 \simeq m_2.$$

We can identify

$$\Delta m^2_\odot \equiv \Delta m^2_{21}, \quad \Delta m^2_{\rm atm} \equiv \Delta m^2_{32} \simeq \Delta m^2_{31},$$

$$|U_{\rm e3}|^2 = \sin^2\theta_{13} < 0.04 \quad ({\rm CHOOZ} + \nu_{\rm A} + \nu_\odot + {\rm KL}),$$

$$|U_{\rm e1}|^2 = \cos^2\theta_\odot(1 - |U_{\rm e3}|^2), \quad |U_{\rm e2}|^2 = \sin^2\theta_\odot(1 - |U_{\rm e3}|^2),$$

$$m_1 \simeq m_2 \simeq \sqrt{|\Delta m^2_{\rm atm}|}.$$

$\cos 2\theta_\odot \gg \sin^2\theta_{13}$: $m_3 \sin^2_{13}|$ negligible in $|<m>|$,

$$|<m>| \cong \sqrt{|\Delta m^2_{\rm atm}|}(1 - s^2_{13})\sqrt{1 - \sin^2 2\theta_\odot \sin^2\left(\frac{\alpha_{21}}{2}\right)},$$

$$\sqrt{|\Delta m^2_{\rm atm}|}\; c^2_{13}|\cos 2\theta_\odot| \leq |<m>| \leq \sqrt{|\Delta m^2_{\rm atm}|}\; c^2_{13}.$$

$$0.01 \text{ eV} \lesssim |<m>| \lesssim 0.055 \text{ eV}.$$

The max, min values: $\alpha_{21} = 0$, $\alpha_{21} = \pm\pi$ - CP-conserving.

$$\sin^2\frac{\alpha_{21}}{2} = \left(1 - \frac{|<m>|^{\;2}}{|\Delta m^2_{\rm atm}|(1 - |U_{\rm e3}|^2)^2}\right)\frac{1}{\sin^2 2\theta_\odot}.$$

Three Quasi-Degenerate Neutrinos

$$m_1 \simeq m_2 \simeq m_3 \equiv m, \quad m^2 \gg |\Delta m^2_{\rm atm}|.$$

We have:

$$\Delta m^2_\odot \equiv \Delta m^2_{21}, \quad \Delta m^2_{\rm atm} \equiv \Delta m^2_{31},$$

$$|U_{\rm e1}|^2 = \cos^2\theta_\odot (1 - |U_{\rm e3}|^2), \quad |U_{\rm e2}|^2 = \sin^2\theta_\odot (1 - |U_{\rm e3}|^2),$$

$$|U_{\rm e3}|^2 = \sin^2\theta_{13} < 0.05 \quad (\text{CHOOZ} + \nu_{\rm A} + \nu_\odot + \text{ KL}).$$

The mass scale m effectively coincides with the $\bar{\nu}_e$ mass $m_{\bar{\nu}_e}$ measured in the current ^{3}H β-decay experiments:

$$m \cong m_{\bar{\nu}_e}.$$

Thus, $m < 2.3$ eV. Cosmology: $m \lesssim (0.7 - 1.8)$ eV.

The QD spectrum - realized for m, which can be measured in the ^{3}H $\beta-$decay experiment KATRIN, $m_{\bar{\nu}_e} \gtrsim (0.2 - 0.3)$ eV.

$$|<m>| \cong m \left| \cos^2\theta_\odot (1 - |U_{\rm e3}|^2) + \sin^2\theta_\odot (1 - |U_{\rm e3}|^2) e^{i\alpha_{21}} + |U_{\rm e3}|^2 e^{i\alpha_{31}} \right|$$

$$\cong m \left| \cos^2\theta_\odot + \sin^2\theta_\odot e^{i\alpha_{21}} \right|;$$

$m\, |\cos 2\theta_\odot| \lesssim |<m>| \lesssim m$; limits: $\alpha_{21} = 0$; $\pm\pi$ - CPC

$$\sin^2 \frac{\alpha_{21}}{2} \cong \left(1 - \frac{|<m>|^{\ 2}}{m(1 - \ |U_{e3}|^2\ )^2}\right) \frac{1}{\sin^2 2\theta_\odot}.$$

Oscillation Parameters

$$\Delta m^2_\odot = 8.0 \times 10^{-5} \text{ eV}^2 \ , \quad 3\sigma(\Delta m^2_\odot) = 12\% \ ,$$

$$\sin^2\theta_\odot = 0.30 \ , \quad 3\sigma(\sin^2\theta_\odot) = 27\% \ ,$$

$$|\Delta m^2_{\text{atm}}| = 2.5 \times 10^{-3} \text{ eV} \ , \quad 3\sigma(|\Delta m^2_{\text{atm}}|) = 28\%.$$

Future:

3 kTy KamLAND: $3\sigma(\Delta m^2_\odot) = 7\%$, $3\sigma(\sin^2\theta_\odot) = 18\%$;

A. Bandyopadhyay et al., hep-ph/0410283

SK-Gd (0.1% Gd: 43×(KL $\bar{\nu}_e$ rate)), 3y: $3\sigma(\Delta m^2_\odot) \cong 4\%$

S. Choubey, S.T.P., hep-ph/0404103;
J. Beacom and M. Vagins, hep-ph/0309300

KL type reactor $\bar{\nu}_e$ detector, $L \sim 60$ km, ~ 60 GW kTy: $3\sigma(\sin^2\theta_\odot) \cong 12\%$

A. Bandyopadhyay et al., hep-ph/0410283 and hep-ph/0302243;
H. Minakata et al., hep-ph/0407326

T2K (SK): $3\sigma(|\Delta m^2_{\text{atm}}|) \cong 6\%$

sgn(Δm^2_{atm}): ν_{atm} experiments, studying the subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations; LBL $\nu-$oscillation experiments (T2K, NOνA); $\nu-$factory.

$\sin^2\theta_{13}$: reactor $\bar{\nu}_e$ experiments, $L \sim (1-2)$ km: Double CHOOZ, Daya-Bay, KASKA,... - factor (5 - 10).

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

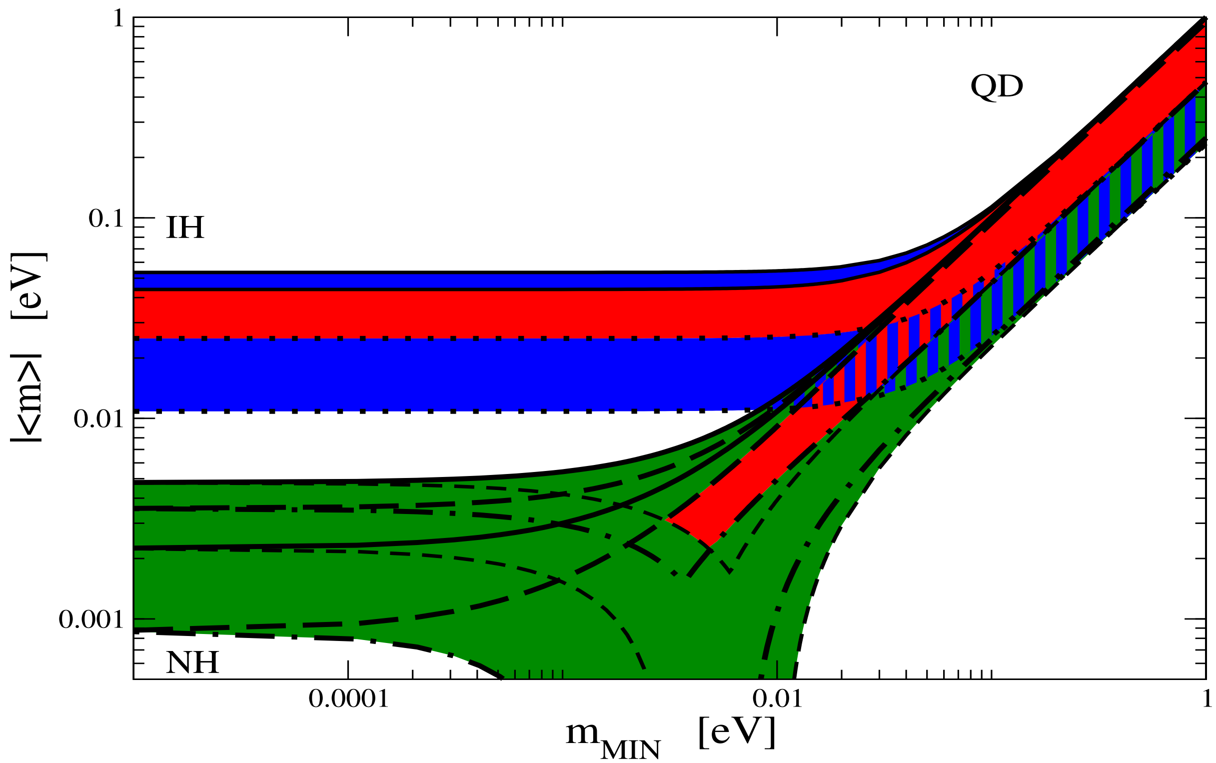
$$\sum_j m_j \equiv \Sigma < (0.4 - 1.7) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

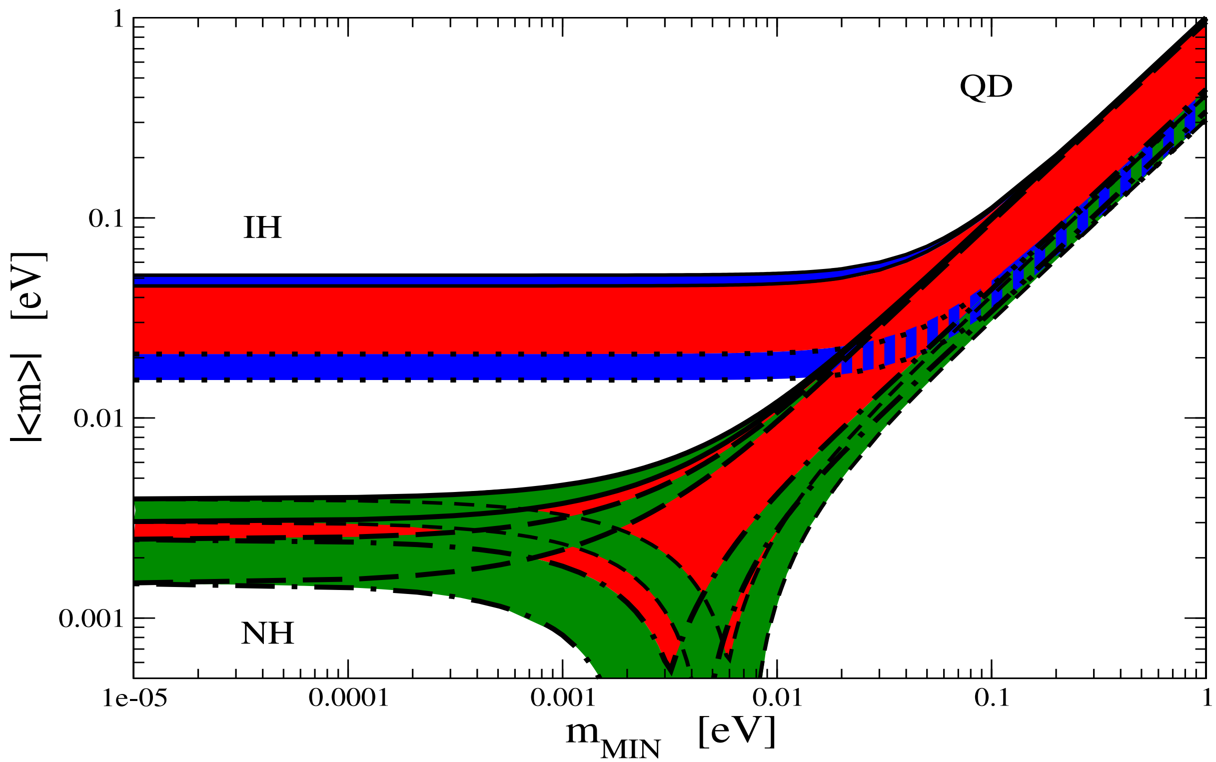
Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$



S. Pascoli, S.T.P., 2006

The current 2σ ranges of values of the parameters used.



S. Pascoli, S.T.P., 2006

$\sin^2\theta_{13} = 0.015 \pm 0.006$; $1\sigma(\Delta m^2_\odot) = 4\%$, $1\sigma(\sin^2\theta_\odot) = 4\%$, $1\sigma(|\Delta m^2_{\rm atm}|) = 6\%$;

$2\sigma(|{<}\, m \,{>}|\ )$ used.

Nuclear Matrix Element Uncertainty

$$|{<}m{>}| = \zeta\ ((|{<}m{>}|_{\rm exp})_{\rm MIN} \pm \Delta)\ ,\quad \zeta \geq 1,$$

$(|{<}m{>}|_{\rm exp})_{\rm MIN}$ – obtained with the **maximal physically allowed value of NME**.

A measurement of the $(\beta\beta)_{0\nu}$-decay half-life time

$$(|{<}m{>}|_{\rm exp})_{\rm MIN} - \Delta \leq |{<}m{>}| \leq \zeta((|{<}m{>}|_{\rm exp})_{\rm MIN} + \Delta)\ .$$

The estimated range of ζ^2:

^{48}Ca, $\zeta^2 \simeq 3.5$

^{76}Ge, $\zeta^2 \simeq 10$

^{82}Se, $\zeta^2 \simeq 10$

^{130}Te, $\zeta^2 \simeq 38.7$

S. Elliot, P. Vogel, 2002

NH vs IH (QD):

$$\zeta\ |{<}m{>}|^{\rm NH}_{\rm max} < |{<}m{>}|^{\rm IH(QD)}_{\rm min}\ ,\ \zeta \geq 1\ .$$

IH vs QD:

$$\zeta\ |{<}m{>}|^{\rm IH}_{\rm max} < |{<}m{>}|^{\rm QD}_{\rm min}\ ,\ \zeta \geq 1\ .$$

S. Pascoli, S.T.P., W. Rodejohann, 2003

Method of Analysis

$$\Gamma_{\rm th} = G\,|\mathcal{M}|^2\,\left(|{<}\,m\,{>}|\;(\mathbf{x})\right)^2\;,\;\;\mathbf{x} = (\mathbf{x}_{\rm osc}, \mathbf{x}^{\mathbf{0}\nu}_{\beta\beta})$$

$$\mathbf{x}_{\rm osc} = (\theta_{\mathbf{12}}, \theta_{\mathbf{13}}, |\mathbf{\Delta m^2_{31}}|, \mathbf{\Delta m^2_{21}})\,,$$

$$\mathbf{x}^{\mathbf{0}\nu}_{\beta\beta} = (m_0, {\rm sgn}(\mathbf{\Delta m^2_{31}}), \alpha_{\mathbf{21}}, \alpha_{\mathbf{31}})\,.$$

$$|{<}\,m\,{>}|^{\;\rm obs} \equiv \sqrt{\frac{\Gamma_{\rm obs}}{G}}\,\frac{1}{|\mathcal{M}_0|}\,,\qquad \sigma_{\beta\beta} = \frac{1}{2}\,\frac{1}{\sqrt{\Gamma_{\rm obs} G}}\,\frac{1}{|\mathcal{M}_0|}\,\sigma(\Gamma_{\rm obs})\,,$$

$|\mathcal{M}_0|$ is some nominal value of the NME.

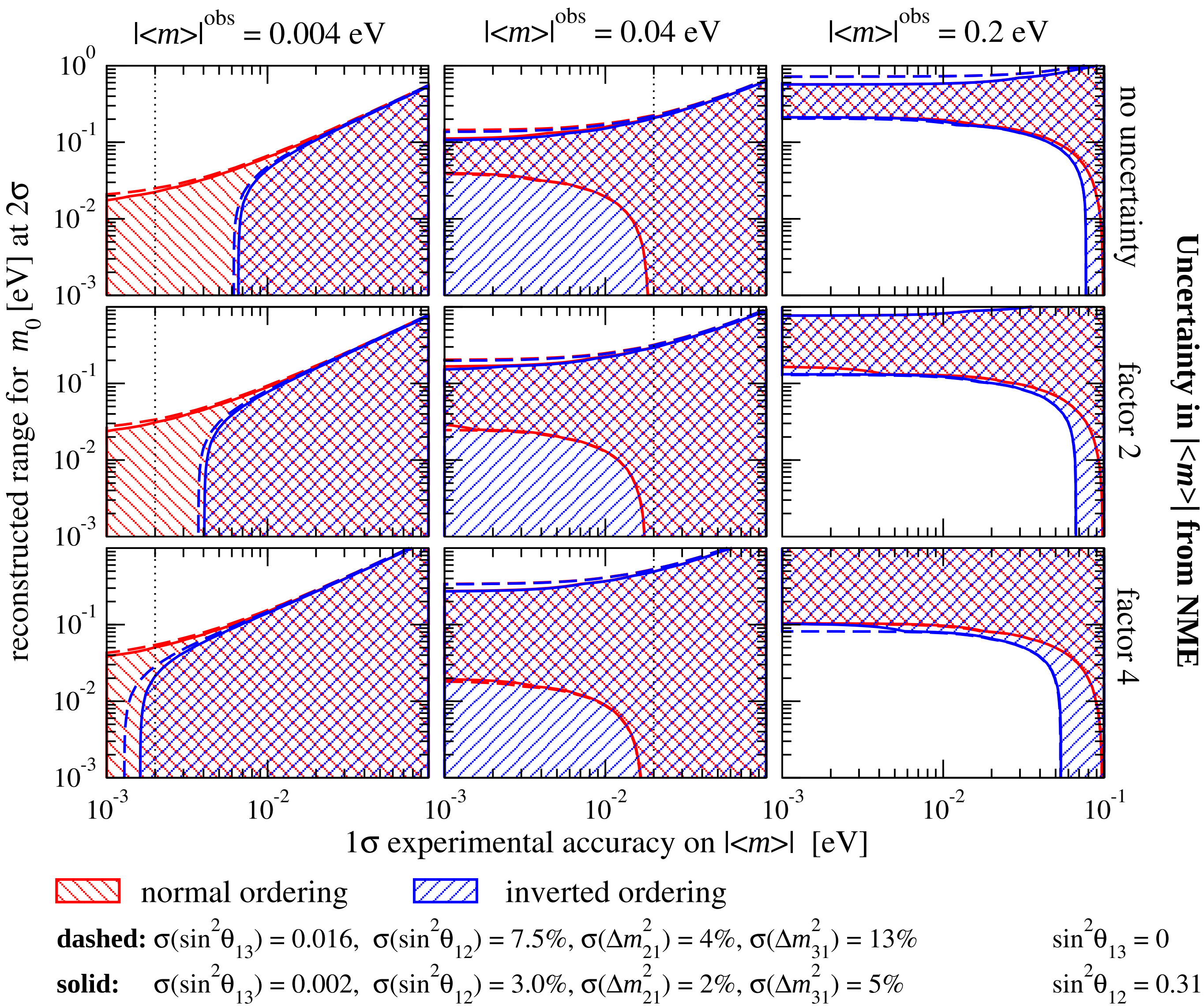
$$\chi^2(\mathbf{x}^{\mathbf{0}\nu}_{\beta\beta}, \mathbf{F}) = \min_{\xi\in[\mathbf{1}/\sqrt{F},\sqrt{F}]} \frac{\left[\xi\,|{<}\,m\,{>}|\;(\mathbf{x}) - |{<}\,m\,{>}|^{\;\rm obs}\right]^2}{\sigma^2_{\beta\beta} + \xi^2\sigma^2_{\rm th}}\,.$$

$$\xi \equiv \frac{|\mathcal{M}|}{|\mathcal{M}_0|}\,,\quad \xi = [1/\sqrt{F}, \sqrt{F}],\quad F \geq 1,$$

$|\mathcal{M}|$ is the *true* value of the NME.

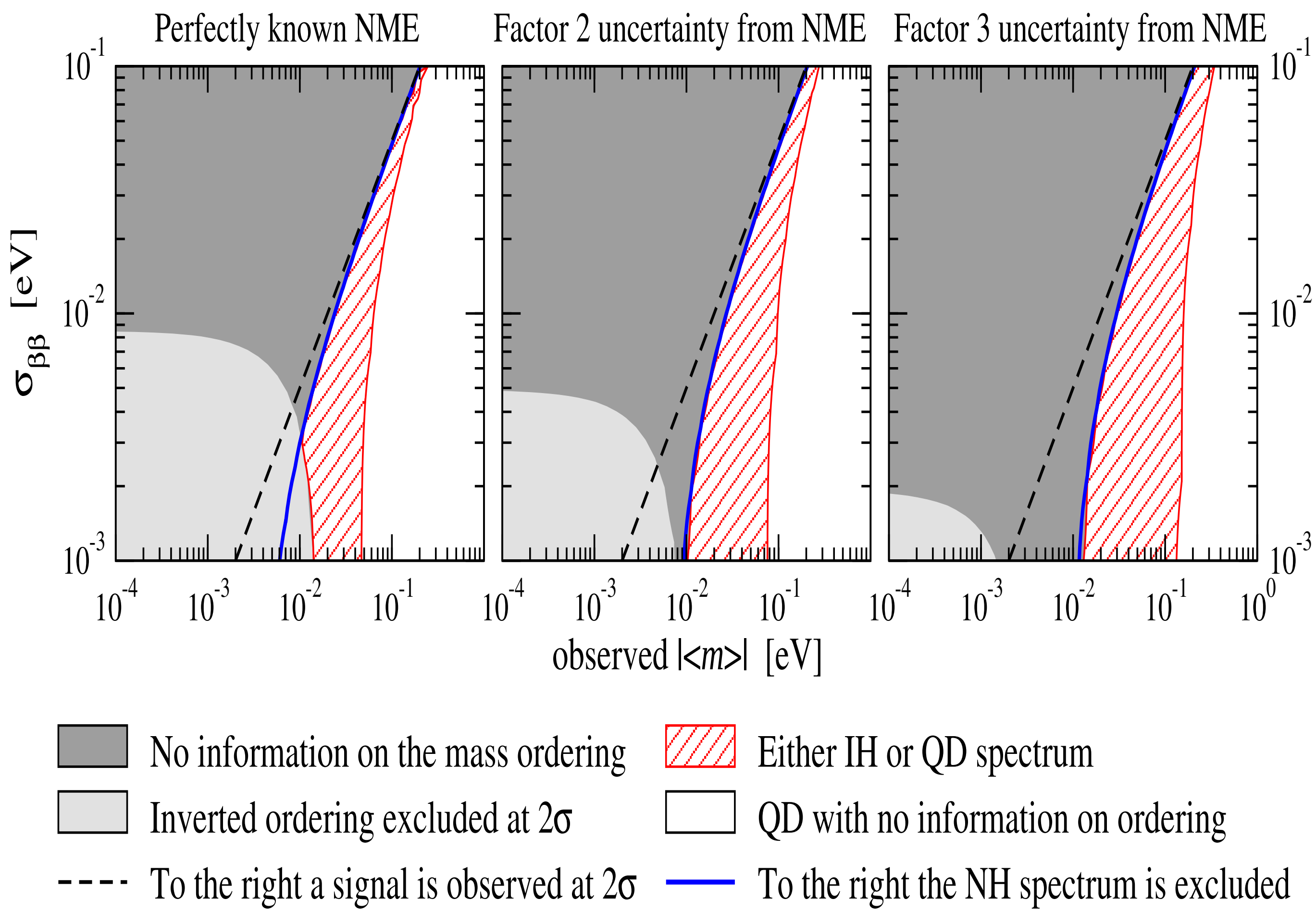
S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

Absolute Neutrino Mass Scale



S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

Distinguishing Between Different Spectra



$\sin^2\theta_{13} = 0.03 \pm 0.006$, $\sin^2\theta_{12} = 0.31 \pm 3\%$, $\Delta m^2_{21} = 8\text{x}10^{-5} \pm 2\%$, $|\Delta m^2_{31}| = 2.2\text{x}10^{-3} \pm 3\%$

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

Majorana CPV Phases and $|<m>|$

IH spectrum: $m_{\rm min} < 0.01$ eV, $\sin^2\theta-$ negligible

$$\sqrt{\Delta m^2_{\rm atm}}|\cos 2\theta_\odot| \leq |<m>| \ \leq \sqrt{\Delta m^2_{\rm atm}}.$$

"Just CP-violating" region:

$$(|<m>|_{\rm exp})_{\rm MAX} < \sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}} \ ,$$

$$(|<m>|_{\rm exp})_{\rm MIN} > \sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}} \ (\cos 2\theta_\odot)_{\rm MAX} \ ,$$

$$|<m>| \ = \zeta \ ((|<m>|_{\rm exp})_{\rm MIN} \pm \Delta) \ , \quad \zeta \geq 1$$

Necessary condition for establishing CP-violation:

$$1 \leq \zeta < \frac{\sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}}}{\sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}} \ (\cos 2\theta_\odot)_{\rm MAX} + 2\Delta} \ \simeq \frac{1}{(\cos 2\theta_\odot)_{\rm MAX}}$$

QD spectrum, $m_{1,2,3} \simeq m_0 \gtrsim 0.20$ eV - similar condition: $\Delta m^2_{\rm atm} \rightarrow m_0^2$.

CPV can be established provided

– $|{<}m{>}|$ measured with $\Delta \lesssim 15\%$;

– $\Delta m^2_{\rm atm}$ (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;

– $\xi \lesssim 1.5$;

– α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;

– $\tan^2\theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

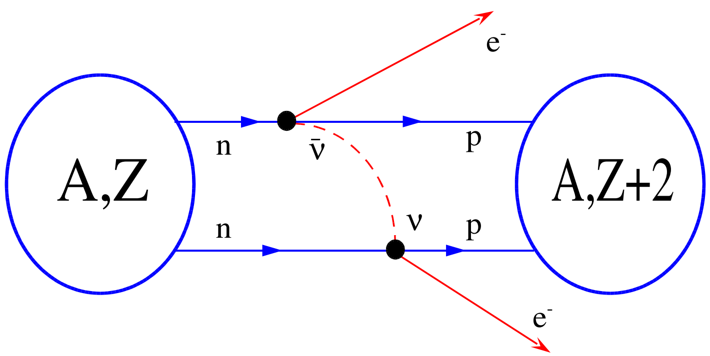
S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

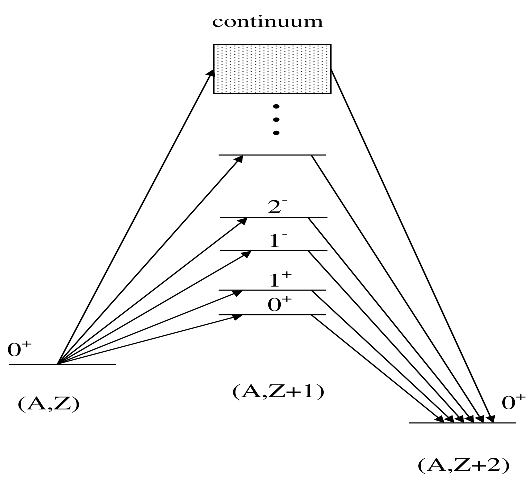
V. Barger *et al.*, 2002

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$dd \rightarrow uue^-e^-(\bar{\nu}_e\bar{\nu}_e)$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

On the NME Uncertainties

The $(\beta\beta)_{0\nu}$-decay half-life

$$(T^{0\,\nu}_{1/2}(A,Z))^{-1} = |{<}m{>}|^{\,2}\,|M^{0\,\nu}(A,Z)|^2\,G^{0\,\nu}(E_0,Z),$$

$G^{0\nu}(E_0,Z)$, E_0 - known phase-space factor and energy release.

If we use a model M of the calculation of NME,

$$|{<}m{>}|^{\,2}_{M}(A,Z) = \frac{1}{T^{0\,\nu}_{1/2}(A,Z)\,|M^{0\,\nu}_{M}(A,Z)|^2\,G^{0\,\nu}(E_0,Z)}.$$

Suppose $(\beta\beta)_{0\nu}-$decay of several nuclei is observed.

$|{<}m{>}|$ cannot depend on parent nucleus (A_j,Z_j).

If the light Majorana ν-exchange - dominant mechanism of $(\beta\beta)_{0\nu}$-decay, **model M for NME can be correct only if**

$$|{<}m{>}|^{\,2}_{M}(A_1,Z_1) \simeq |{<}m{>}|^{\,2}_{M}(A_2,Z_2) = ...$$

For different models and the same nucleus (A,Z),

$$|{<}m{>}|^{\,2}_{M_1}(A,Z)\ |M^{0\nu}_{M_1}(A,Z)|^2 = |{<}m{>}|^{\,2}_{M_2}(A,Z)\ |M^{0\nu}_{M_2}(A,Z)|^2 = ...,$$

$$|{<}m{>}|^{\,2}_{M_2}(A,Z) = \eta^{M_2;M_1}(A,Z)\,|{<}m{>}|^{\,2}_{M_1}(A,Z)\ ,$$

$$\eta^{M_2;M_1}(A,Z) = \frac{|M^{0\,\nu}_{M_1}(A,Z)|^2}{|M^{0\,\nu}_{M_2}(A,Z)|^2}\ .$$

Nucleus	$\eta^{M_2;M_1}$	$\eta^{M_3;M_1}$	$\eta^{M_2;M_3}$
^{76}Ge	0.37	0.19	1.93
^{82}Se	—	0.38	—
^{100}Mo	—	—	6.56
^{130}Te	0.74	0.10	7.32
^{136}Xe	0.53	0.02	22.42

M_1 (SM): E. Caurier et al., 1999; M_2 (QRPA): V. Rodin et al., 2003;
M_3 (QRPA): O. Civatarese and J. Suhonen, 2003.

The observation of $(\beta\beta)_{0\nu}$-decay of at least 3 nuclei would be important for the solution of the problem of NME.

Table 2 suggests: ^{76}Ge, ^{130}Te, ^{136}Xe .

If for some model M

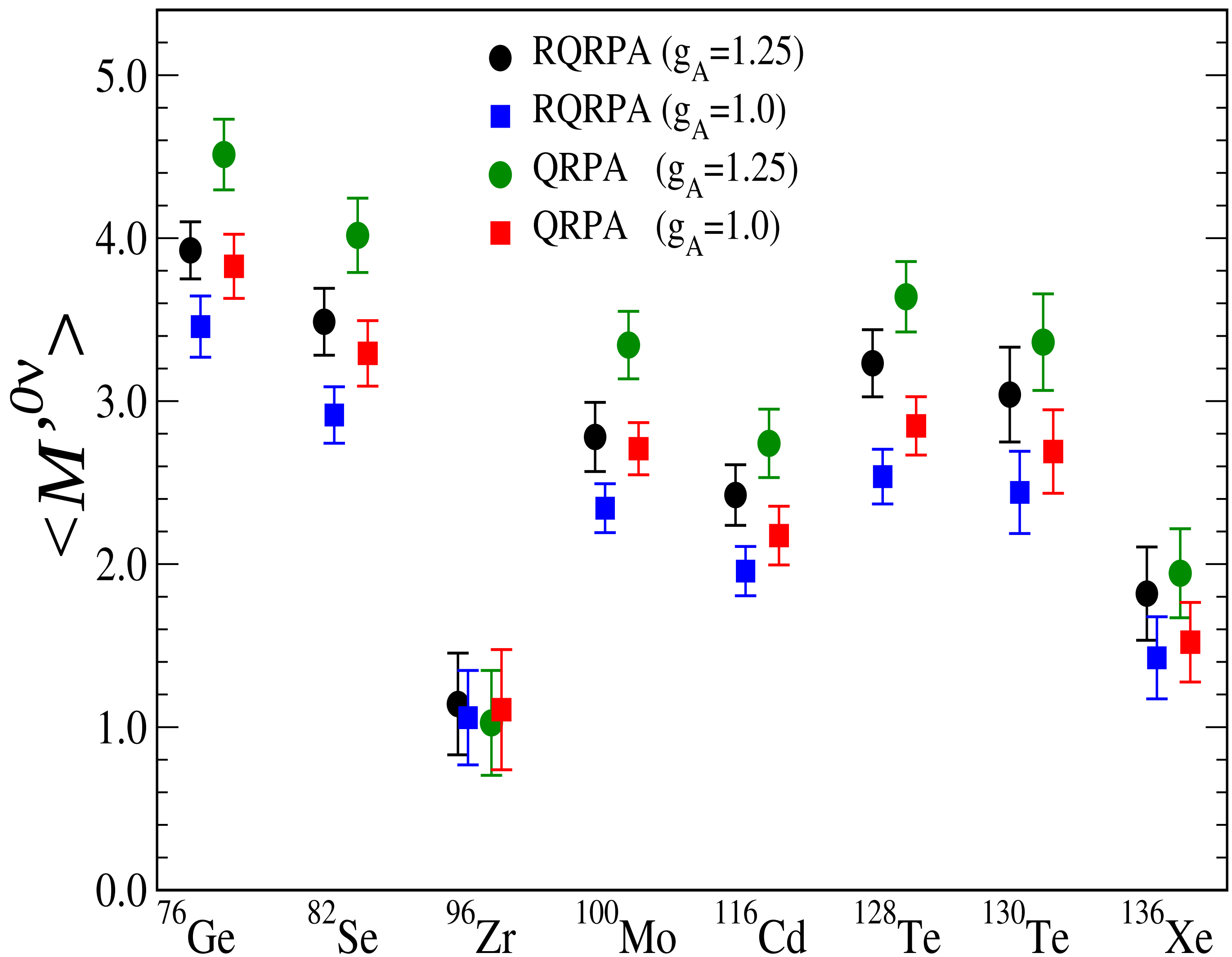
$$|<m>|\,^2_M(A_1,Z_1) \simeq |<m>|\,^2_M(A_2,Z_2) = ... \equiv |<m>|\,^2_0 \,,$$

$|<m>|\,_0$ - the true value (most likely).
Strong dependence of NME on (A,Z) - crucial for the test.

S. M. Bilenky, S.T.P., 2004

Encouraging results on the problem of calculating the NME $(\xi \lesssim 1.5)$ have been obtained recently in

V. A. Rodin, A. Faessler, F. Simkovic, P. Vogel, nucl-th/0503063



V. A. Rodin *et al.*, nucl-th/0503063

The errors have no statistical origin, just illustrate the degree of the variation of the results by changing the basis size. The "systematic error" of the QRPA (due to neglecting many-particle configurations): $(3 \div 5) \times 10\%$, can vary from one nucleus to another.

Alternative Mechanisms of $(\beta\beta)_{0\nu}$-Decay

- Light neutrino exchange

- R-parity violating SUSY

- Heavy neutrino exchange

- Right-handed weak currents

Conclusions

$(\beta\beta)_{0\nu}$-decay experiments have remarkable physics potential:

– Can establish the Majorana nature of ν_j

– Can provide unique information on the ν mass spectrum

– Can provide unique information on the absolute scale of ν masses

– Can provide information on the Majorana CPV phases

The knowledge of the values of the relevant $(\beta\beta)_{0\nu}$–decay NME with a sufficiently small uncertainty is crucial for obtaining quantitative information on the neutrino mass and mixing parameters from a measurement of $\Gamma(\beta\beta)_{0\nu}$.

The precision in the measurement of $\Gamma(\beta\beta)_{0\nu}$ will also be very important for the quantitative interpretation of the data.